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## Lafayette, Indiana

Improved Fluid Dynamics Similarity,
Analysis and Verification

Final Report - Part IV
NAS 8-20222
ACOUSTIC VELOCITIES IN TWO-PHASE MIXTURES
OF SOME CRYOGENIC FLUIDS

by

E. I. Griggs, E. R. F. Winter
R. J. Schoenhals

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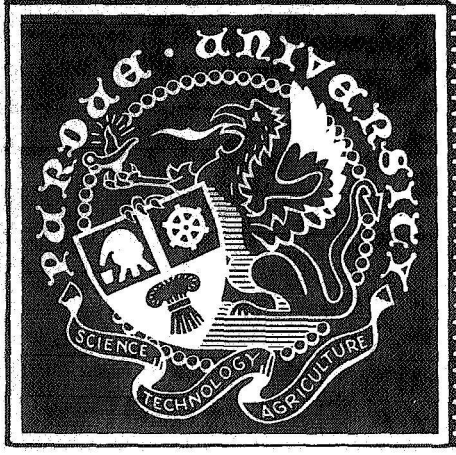


## School of Mechanical Engineering
## Heat Transfer Laboratory

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Submitted by

Heat Transfer Laboratory
School of Mechanical Engineering
Purdue University
Lafayette, Indiana

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## ABSTRACT

Calculated values of the acoustic velocity are presented graphically for single-component and two-component two-phase mixtures. Effects of void fraction or quality, temperature, and system pressure are illustrated.

Because of the inherent vapor formation in cryogenic equipment, and due to the importance of two-phase conditions in space applications, acoustic velocity values were determined for two-phase mixtures of some of the cryogenic fluids used in propulsion systems of space vehicles. This was prompted by consideration of previously published information on acoustic velocities of two-phase fluids which demonstrated the need for further study.

Three different analytical models were employed. Model 1, which incorporates an assumption of thermodynamic equilibrium, is applicable only to single-component two-phase mixtures. Models 2 and 3, which involve an assumption that the pressure disturbance travels through the medium without any change in the relative masses of the two phases, are primarily applicable to two-component two-phase mixtures. However, for purposes of comparison, all three models were used in making calculations for single-component, two-phase mixtures of oxygen, nitrogen, and parahydrogen. Model 2 was used for making calculations for two-component, two-phase mixtures of oxygen and helium and of hydrogen and helium. In all cases, the results are shown graphically so that the effects of variations in quality or void fraction, temperature, and pressure are illustrated.

The acoustic velocity values presented for two-component mixtures

should be useful for design purposes. Deviations in the various values for single-component mixtures dictate a need for experimental work in this area to determine the analytical model which best describes the actual physical behavior.

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# NOMENCLATURE

| Symbol | Description | Dimensions |
|---|---|---|
| A | cross-sectional area | $L^2$ |
| C | acoustic velocity | $LT^{-1}$ |
| $C_f$ | frictional influence coefficient | $FL^{-1}$ |
| $C_x$ | phase change influence coefficient | F |
| $C_A$ | area change influence coefficient | F |
| $C_g$ | gravity influence coefficient | $FL^{-2}T^2$ |
| $C_v$ | specific heat at constant volume | $FLM^{-1}\theta^{-1}$ |
| $C_p$ | specific heat at constant pressure | $FLM^{-1}\theta^{-1}$ |
| F | denotes dimension of force | |
| g | acceleration of gravity | $LT^{-2}$ |
| $g_c$ | dimensional constant | $MLF^{-1}T^{-2}$ |
| K | specific heat ratio | |
| L | denotes dimension of length | |
| M | Mach number | |
| M | denotes dimension of mass | |
| m | mass | M |
| P | pressure | $FL^{-2}$ |
| R | ideal gas constant | $FLM^{-1}\theta^{-1}$ |
| s | entropy per unit mass | $FLM^{-1}\theta^{-1}$ |
| T | temperature | $\theta$ |
| V | volume | $L^3$ |
| v | specific volume | $L^3M^{-1}$ |
| x | quality, ratio of mass of gaseous phase to total mass of mixture | |

# NOMENCLATURE

| Symbol | Description | Dimensions |
|---|---|---|
| z | coordinate in direction of flow | L |
| $\alpha$ | void fraction | |
| $\beta$ | isentropic compressibility | $L^2F^{-1}$ |
| $\rho$ | density | $ML^{-3}$ |
| $\phi$ | ratio of mass of gaseous phase to mass of liquid phase | |
| $\theta$ | angle between direction of flow and gravity | |
| $\Theta$ | denotes dimension of temperature | |

## Subscripts

| | |
|---|---|
| 1 | liquid phase |
| 2 | gas or vapor phase |
| b | exhaust pressure |
| O | stagnation value |
| s | constant entropy differentiation |

## Superscripts

| | |
|---|---|
| $\bar{}$ | denotes average value |

# INTRODUCTION

In recent years extensive attention has been focused on the study of two-phase flow, a fact clearly exemplified by the expanding number of publications appearing in this area. Cooling towers, steam turbines, and various chemical processes are typical examples of applications where the simultaneous flow of a liquid and a gas or a liquid and its vapor occur. The advent of nuclear reactors served to further promote interest in two-phase flow phenomena. Because of the almost inherent phase changes of cryogenic fluids due to heat transfer, cryogenic engineering has also attracted considerable attention to two-phase flow. Properties of cryogenic fluids have been gaining even higher levels of importance as a result of the utilization of these fluids in the area of space exploration. A propellant supply system is one particular appplication in this area where two-phase mixtures may occur due to heat transfer as well as cavitation.

Pressure drop, heat transfer characteristics, and flow rate predictions are typical engineering problems which become more complicated with the appearance of two phases. The mixing of a relatively small amount of gas or vapor with a liquid changes an initially incompressible situation into a compressible one; consequently, the study of two-phase mixtures leads directly to those parameters which characterize compressible flow. One of the significant parameters useful in understanding the nature of single-phase compressible flow is the acoustic velocity. This suggests that the acoustic velocity may have a similar prominence in the study of two-phase mixtures.

The present investigation was undertaken for the purpose of calculating the acoustic velocities of two-phase mixtures of certain cryogenic fluids using models which have already been discussed in the literature for the case of water mixed with various gases. The results of the computations were to serve as useful data in the more comprehensive study of the flow characteristics of propellant supply systems. In particular, acoustic velocities were thought to be of significant value in the study of choked flow which is discussed in more detail in a subsequent section of this study and part III of this final report.

Presented in this report are the results of the computations of acoustic velocities for two-phase mixtures of pure nitrogen, oxygen, and parahydrogen as well as the results of computations using a particular model for oxygen-helium and hydrogen-helium mixtures. Three computational models are compared for single-component mixtures of nitrogen, oxygen, and parahydrogen. The results emphasize the need for additional experimentation in the determination of acoustic velocities in two-phase mixtures in order to provide better understanding of the physics of two-phase flow situations. This would be a prerequisite to better design of systems where acoustic velocity data is fundamental.

## CRITICAL FLOW OF TWO-PHASE MIXTURES

One of the interesting as well as important features associated with single-phase compressible flow is the phenomenon of choking.

For specified inlet conditions and fixed flow-passage geometry, analyses show that the flow rate increases as the exhaust pressure decreases until ultimately a maximum flow rate is achieved. When this condition occurs, additional decreasing of the exhaust pressure does not affect the flow rate. The flow, therefore, is said to be choked, a condition also referred to in the literature as critical flow or mass-limiting flow. A qualitative illustration of this type of behavior is presented in Figure 1. An exact quantitative representation depends upon the particular flow situation. For example, the pressure ratio at which the flow rate first reaches its maximum value for the case of an ideal gas flowing isentropically through a converging passage depends only upon the ratio of the specific heats of the gas. For the case of an ideal gas flowing adiabatically through a constant area pipe with friction, the pressure ratio at which the flow rate first reaches its maximum value is a function of the friction factor as well as the geometry. Extensive treatment of this type of gas behavior can be found in various texts on compressible fluid flow such as the one by Shapiro(1)*

When the contributive conditions yield critical flow of a single-phase fluid, a salient feature is the existence of a Mach number of unity. The Mach number is defined as the ratio of the fluid velocity at a point to the acoustic velocity in the fluid at the same point; consequently, critical flow becomes an important consideration when actual fluid velocities are of the same

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*Underlined numbers in parentheses designate references listed at the end of this report.

order of magnitude as the acoustic velocity of the fluid. For a converging passage this condition occurs at the exit while for a converging-diverging passage this condition occurs at the minimum area. The emergence of the acoustic velocity in the study of critical flow is one example of the significance of this parameter in the general field of compressible flow.

For an incompressible medium, the acoustic velocity is infinite. Although water is usually assumed incompressible in many analyses, it does exhibit a finite compressibility which results in an acoustic velocity of approximately 5000 ft/sec. For air at room temperature the acoustic velocity is around 1120 ft/sec. Considering water-air as a typical two-phase mixture, it might appear that practical applications involving two-phase mixtures would never involve real velocities anywhere near the value of the acoustic velocity in the gaseous phase alone and, consequently, critical flow would not be of immediate concern. It is in this regard, however, that surprising results have been discovered. In a survey (2) of the acoustic velocities of two-phase mixtures, significantly low values of acoustic velocities are reported and discussed.

Treatment of two-phase flow is complicated by the possibility of a variety of flow patterns. Figures 2 and 3, which come from reference (3), illustrate several patterns. Clearly, the validity of the results of any study strongly depends on how accurately the flow mechanism can be described. The phase interactions which produce the respective flow pattern lead to definite analytical difficulties. In the literature considerable effort has been devoted to the establishment of an analytical model which can be

effectively utilized in making dependable predictions; however, the assumptions built into a model usually limit, quite severly in some cases, its applicability. Four models which have been employed in critical flow analyses are briefly described below. The first three are described in reference (4) and the fourth is described in reference (5).

1. Homogeneous, Thermal Equilibrium Model - The two phases are assumed to be homogeneously mixed with uniform distribution of the phases and thermal equilibrium is assumed to exist throughout the process.
2. Separated-Phase, Vapor Choking Model - The two phases are assumed to be flowing separately with uniform temperature at each cross section, and the criterion for choking is considered to be the condition when the vapor attains sonic velocity.
3. Homogeneous, Metastable Model - The two phases are assumed homogeneous with uniform pressure and temperature, but time has not permitted any phase change to occur.
4. Fauske's Slip Model - An annular type flow is assumed with the liquid and vapor, which are assumed to be in equilibrium, flowing at uniform but separate velocities.

With regard to cryogenic propellant systems, the importance of critical flow is discussed in reference (12) which also includes comparisons of experimental data for critical flow of two-phase nitrogen with certain theoretical models.

Some insight into the correlation between critical flow, acoustic velocity, and the physical model may be gained by consideration

of a brief mathematical description. A one-dimensional steady state analysis (3) will eventually lead to an expression for the pressure gradient of the form:

$$-\frac{dP}{dz} = \frac{C_f + C_x \frac{dx}{dz} + C_A \frac{1}{A}\frac{dA}{dz} + C_g g_1 \cos\theta}{1 - M^2} \qquad (1)$$

The terms in the numerator on the right-hand side of equation (1) represent factors which contribute to the pressure variation in the direction of flow. The influence of friction, phase change, area change, and gravity on the pressure gradient is expressed, respectively, by $C_f$, $C_x$, $C_A$, and $C_g$. These coefficients are called influence coefficients. The term $M^2$ in the denominator may be interpreted to have the same significance as the Mach number in single-phase flow since critical flow corresponds to an infinite pressure gradient which results in equation (1) when $M^2$ is unity. Accordingly, an expression for an equivalent acoustic velocity in a two-phase mixture may be deducted from an expression for the pressure gradient. This, however, depends strongly on the physical flow pattern which is prevalent in a given situation. For example, the frictional contribution in equation (1) will strongly depend on the dispersion of the two phases. The possible extremes are indicated in Figures 2 and 3. In a single-component situation the phase change term may be very significant while in a two-component situation the contribution of the phase change term may be very small.

For the case of distinct phase separation, a pressure disturbance would probably travel through one phase at a different

velocity than it would travel through the other phase. Relative dispersion of phases would also contribute to attenuation of the disturbance as well as causing other effects such as scattering. Consequently, references in this report to the acoustic velocity should be interpreted from the viewpoint that the two-phase mixture is a pseudo-fluid. This implies that the mixture is a homogeneous medium with uniformly dispersed phases such that an average density has meaning throughout the system on a differential basis and, accordingly, the medium behaves as a single-phase fluid possessing a density equal to the average density of the mixture.

## ACOUSTIC VELOCITY CALCULATIONS

### Physical Model

For the calculations reported in this work, the two-phase mixture is assumed to be a pseudo-fluid as mentioned previously. The medium is assumed to be homogeneous with the phases uniformly dispersed such that mean or average densitites have meaning on a differential basis. The behavior of the medium is assumed to be such that acoustic velocities have significance as the speed of an infinitesimal pressure disturbance whereas effects such as dispersion, scattering, and attentuation are not considered.

### Basic Definitions

For the two-phase mixture, properties of the gaseous or vapor phase are denoted with a subscript 2 while those for the liquid

phase are denoted with a subscript 1. For the case of a single component mixture under the restriction of thermodynamic equilibrium, these then denote saturation properties.

Quality is defined as the mass fraction of the mixture which is in the gaseous phase.

$$x = \frac{m_2}{m} = \frac{m_2}{m_1 + m_2} \tag{2}$$

Similarly, the void fraction is defined as a volume fraction.

$$\alpha = \frac{V_2}{V} = \frac{V_2}{V_1 + V_2} \tag{3}$$

The average specific volume can be expressed as:

$$\bar{v} = xv_2 + (1-x)v_1 \tag{4}$$

The average density can be expressed as:

$$\bar{\rho} = \alpha\rho_2 + (1-\alpha)\rho_1 \tag{5}$$

Analysis of the velocity of propagation of an infinitesimal pressure disturbance through a pure substance yields (1):

$$C^2 = g_c\left(\frac{\partial P}{\partial \rho}\right)_s = -v^2 g_c\left(\frac{\partial P}{\partial v}\right)_s \tag{6}$$

The isentropic compressibility is defined as:

$$\beta = -\frac{1}{v}\left(\frac{\partial v}{\partial P}\right)_s \tag{7}$$

Therefore, combination of equations (6) and (7) yields:

$$c^2 = \frac{g_c}{\beta\rho} \tag{8}$$

For an ideal gas, equation (6) gives:

$$c^2 = Kg_cRT \tag{9}$$

Computational Models

The two cases, single-component and two-component two-phase mixtures, which are considered in this report are discussed quite extensively in reference (2). A detailed analysis and experimental study of the propagation of pressure pulses through a mixture of steam and water is reported in reference (6). The results presented in reference (2) for a two-component mixture are for water mixed with various gases. The purpose of this study was to utilize the available models to calculate values for certain cryogenic fluids that might be used in propellant supply systems.

Direct evaluation of acoustic velocities could be made using equation (6) provided an appropriate density, pressure, and entropy relationship could be written; however, this entails dealing with the complexities of a valid equation of state.

Model 1

The first model considered was based on the computation procedure for a single-component two-phase mixture utilized in reference (6) for water and steam. Equation (6) was recast in terms of saturation properties and derivaties of saturation properties.

The derivativies were approximated with finite differences using tabulated saturation values.

For the assumptions of a homogeneous mixture in thermodynamic equilibrium, the average specific volume given by equation (4) was used in equation (6) to give:

$$C^2 = \frac{g_c\,[v_1 + x(v_2-v_1)]^2}{x\left[\left(\frac{v_2-v_1}{s_2-s_1}\right)\cdot\left(\frac{ds_2}{dP}-\frac{ds_1}{dP}\right)-\left(\frac{dv_2}{dP}-\frac{dv_1}{dP}\right)\right] + \left(\frac{v_2-v_1}{s_2-s_1}\right)\cdot\frac{ds_1}{dP} - \frac{dv_1}{dP}} \qquad (10)$$

This equation involves only saturation properties and their derivatives with respect to pressure. The derivation of equation (10) is included in the Appendix.

Model 2

The second model considered was primarily applicable to two-component two-phase mixtures but was also used for some single-component mixtures for comparison purposes. Basically, the model is based on the assumptions that the gaseous phase is an ideal gas, the liquid has a constant isentropic compressibility, the quality remains constant, and the pressure disturbance is isentropic. In reference (2), where the analytical derivation is presented, results are given for mixtures of water with various gases. In those calculations assumptions that the vapor pressure of the liquid was negligible and that the gas was insoluble in the liquid were also included.

The equation used for computations was

$$\left(\frac{C}{C_2}\right)^2 = \frac{\left(\frac{\rho_2}{\rho_1}+\phi\right)^2\left(\frac{C_{v1}}{K_2C_{v2}}+\phi\right)}{\phi(1+\phi)\left(\frac{C_{v1}}{C_{v2}}+\phi\right)\left(1+\frac{P\beta_1\rho_2}{\phi\rho_1}\right)} \qquad (11)$$

where

$$\phi = \frac{x}{1-x} \tag{12}$$

The derivation of equation (11) is also included in the Appendix.

Model 3

The third model follows from reference (3). An expression for the speed of a compressibility wave was deduced from a relation for the pressure gradient. As mentioned earlier, this may be regarded as an effective acoustic velocity. The same expression can be obtained from equation (6) provided the differentiation is interpreted properly. This is included in the Appendix. In either case the model is based on the assumption that the pressure disturbance travels through the mixture without any phase change. The expression which was used for calculations is:

$$\frac{1}{c^2} = [\alpha\rho_2 + (1-\alpha)\rho_1] \cdot [\frac{\alpha}{c^2_2\rho_2} + \frac{(1-\alpha)}{c^2_1\rho_1}] \tag{13}$$

Computational Procedures:

For each model computer programs were written to perform the necessary calculations as a function of quality or void fraction. For each temperature or pressure, salient data were supplied as input to the programs.

Model 1

Use of equation (10) required extensive saturation data. The saturation data which was finally acquired was tabulated in

convenient increments of temperature rather than pressure; therefore, in each case derivatives with respect to pressure were obtained from

$$\frac{dv_1}{dP} = \frac{dv_1}{dT}\frac{dT}{dP} \tag{14}$$

where $v_1$ is representive of any property involved. The derivatives with respect to temperature were approximated by finite differences using tabulated values.

$$\frac{dv_1}{dT} \simeq \frac{\Delta v_1}{\Delta T} \tag{15}$$

For nitrogen and oxygen, values of $dT/_{dP}$ were obtained by differentiation of the vapor pressure equation. For nitrogen, saturation properties and the vapor pressure equation were obtained from reference (8). For oxygen, similar quantities were obtained from reference (7). For parahydrogen $dT/_{dP}$ was evaluated using Clapeyron's equation.

$$\frac{dT}{dP} = \frac{v_2 - v_1}{s_2 - s_1} \tag{16}$$

Saturation values were obtained from reference (9).

Model 2

Equation (11) is written to yield a ratio of the acoustic velocity of the mixture to the acoustic velocity of phase 2. Since phase 2 is assumed to be an ideal gas, its acoustic velocity is given by equation (9).

In reference (2), equation (11) was used to calculate values for mixtures of water with various gases. The vapor pressure of the water was assumed to be negligible in each case compared to the total pressure. For this report, calculations were made for oxygen-helium and hydrogen-helium mixtures. For the temperature and pressure values considered, the vapor pressure of the liquid appeared to be significant; therefore, phase 2 was still assumed to be an ideal gas having properties indicative of an ideal gas mixture of the two constituents. The partial pressure of the vapor in the mixture was taken as the saturation pressure for the specified temperature. Specific heat values were obtained from reference (10). Values for the isentropic compressibility were obtained indirectly from reference (11) by using values of the acoustic velocity of the liquid together with equation (8).

Model 3

Densities and acoustic velocities of the separate phases were required for calculations using equation (13). Density values for nitrogen and oxygen were obtained from references (8) and (7), respectively. Acoustic velocities for the respective phases of nitrogen and oxygen were obtained from reference (11). An extensive range of properties for parahydrogen were available in reference (9). This permitted calculations for parahydrogen over a wider range of conditions.

## DISCUSSION OF RESULTS

The results of the calculations by Model 1 for single component two-phase mixtures of nitrogen, oxygen, and parahydrogen

are presented in Figures 4 through 9. In Figures 4, 6, and 8 acoustic velocities are plotted as a function of saturation pressure for various qualities. In Figures 5, 7, and 9 they are plotted as a function of quality for certain saturation pressures. For a given saturation pressure, the acoustic velocity increases with quality. Very small values are seen to exist near the saturated liquid line in the two-phase region for all three substances. This behavior is due to the assumption of thermodynamic equilibrium incorporated in model 1. As discussed in reference (6), a small pressure change causes a relatively large change in the mean density as long as some vapor is present. Because of the relatively large values of acoustic velocity in the pure liquid phase, this model suggests a discontinuity in the acoustic velocity at the saturated liquid line.

Figures 10, 11, and 12 illustrate the results from all three models for single component two-phase mixtures of nitrogen, oxygen, and parahydrogen respectively. In these figures values of acoustic velocity are plotted as a function of void fraction for saturation conditions near one atmosphere pressure. Models 2 and 3 yield similar trends. For void fraction values less than 2 per cent the acoustic velocity drops very rapidly from its value in the liquid. A minimum value is predicted in the vincinity of a void fraction of 50 per cent. Model 2, which incorporates the assumption that the vapor is an ideal gas, yields lower values than does model 3 over most of the range of void fraction; however, the curve for model 2 crosses that for model 1 at higher

void fractions. Figure 13 compares all three models for parahydrogen at a considerably higher pressure. The void fraction value where the curves cross is seen to decrease with an increase in pressure. This characteristic is probably due to the ideal gas assumption.

While the percentage error between models 2 and 3 is significant over most of the range of void fraction, the deviation of model 1 from models 2 and 3 is remarkably large and increases with a decrease in void fraction. This illustrates quite vividly the influence of the assumptions. Models 2 and 3 omit the possibility for any phase change while model 1 invokes the necessary phase change to permit thermodynamic equilibrium to exist.

Figure 14 illustrates the influence of saturation pressure and temperature on the calculations by model 3 for parahydrogen. For the liquid, acoustic velocity values decrease with an increase in temperature and pressure. But over most of the range of void fraction, the values increase with temperature and pressure.

Results for two-component two-phase mixtures of oxygen and helium are given in Figures 15 through 19. These were obtained using model 2. Figures 15 through 18 illustrate the influence of pressure for fixed temperatures. For a given temperature and void fraction the acoustic velocity increases with pressure. Figure 19 illustrates a slight temperature influence for a given pressure. Similar behavior is illustrated in Figures 20 through 22 for hydrogen-helium mixtures. The difference in behavior at a void fraction near 100 per cent for the oxygen-helium mixture

shown in Figure 19 compared to that for the hydrogen-helium mixture shown in figure 22 can be explained by the fact that model 2 incorporates the assumption that phase 2 is an ideal gas mixture of the two constituents. Because of the relative magnitudes of the molecular weights of oxygen, hydrogen, and helium, a temperature increase has a larger influence on decreasing the gas constant for the oxygen-helium mixture than it does for the hydrogen-helium mixture. Equation (9) shows that a decrease in the gas constant counteracts the influence of an increase in temperature on the acoustic velocity. The characteristic minimum near 50 per cent void fraction is displayed in all cases.

## CONCLUSIONS

Precise acoustic velocity values are needed in two-phase compressible flow work in order to support dependable design calculations. Calculated values of acoustic velocities for nitrogen, oxygen, and parahydrogen given in this report demonstrate large variations depending on the model employed for the computations. Near the saturated vapor line all three models predict reasonable values; however, near the saturated liquid line, the model which invokes thermodynamic equilibrium yields extremely low values. In this latter region, acoustic velocity values are predicted that are lower than those in the liquid by factors close to 300. This feature vividly illustrates the importance of understanding the physics associated with two-phase flow.

In reference (2), it is reported that experimental results available for water-air mixtures agree well with the predictions for two-component two-phase mixtures. Model 2 in this report corresponds to analytical predictions reported there. However, additional experimentation is needed for two-component two-phase mixtures where the vapor pressure is significant compared to the total pressure.

While investigations of critical flow have been performed, the need for additional study is apparent, especially in the related area of acoustic velocity. Additional correlation of the characteristics of various flow mechanisms would benefit both interests. With regard to critical flow, it was recommended in reference (4) that the vapor choking model be used at higher qualities and the

homogeneous models be used as upper and lower limits for variable area flow at low qualities. With regard to acoustic velocities, the homogeneous models studied in this report yield upper and lower bounds at low qualities that are separated by several hundred per cent. Certainly, experimental effort is needed in this area.

There appear to be no direct experimental measurements available for acoustic velocities in single-component two-phase mixtures. Additional experimentation is certainly needed to correlate acoustic velocities with various two-phase flow patterns and their respective characteristics. Such data is especially needed in single-component two-phase studies where the contribution of phase change is prevalent and, according to the comparisons made in this report, quite large.

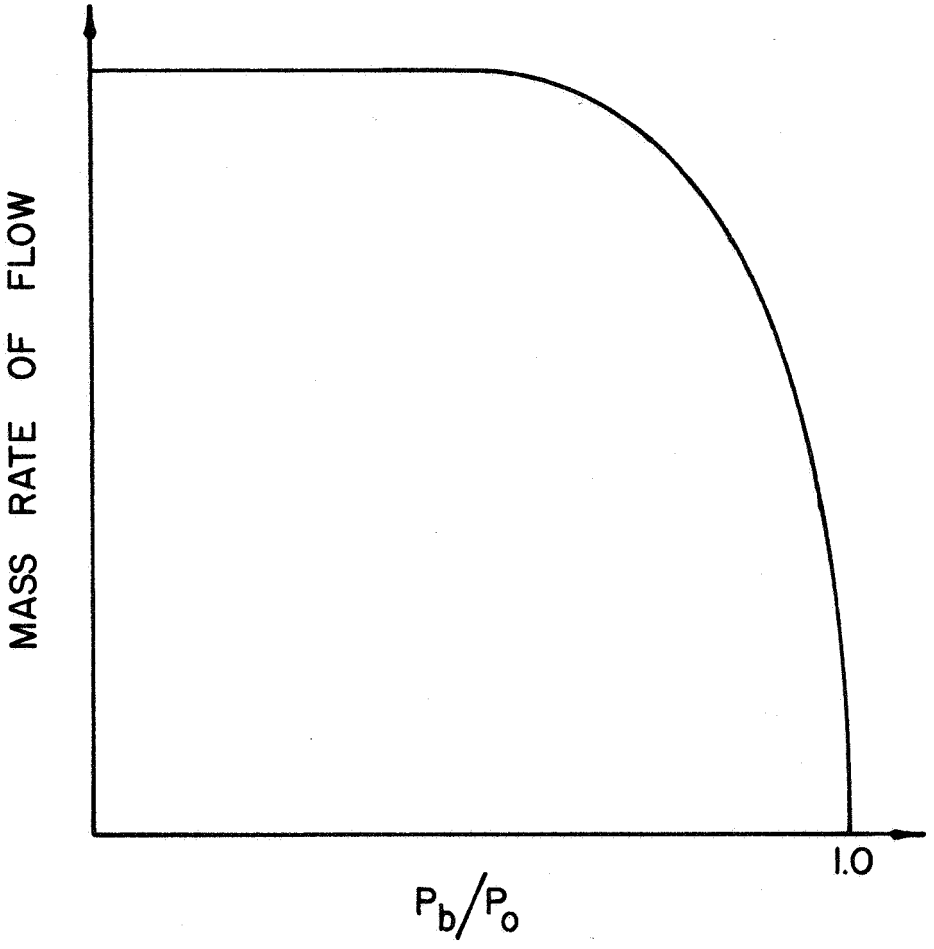


Figure 1: Schematic Representation of Choked Flow

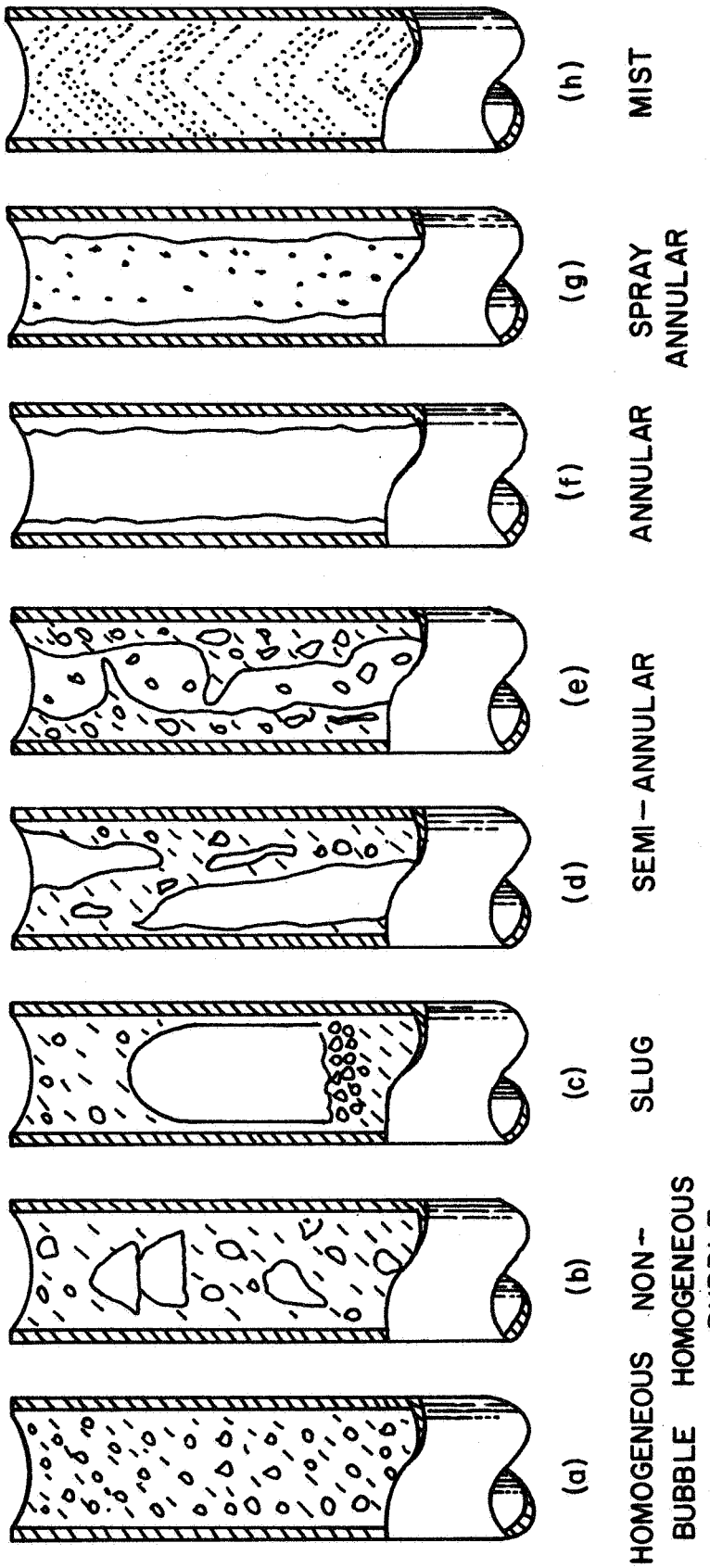


Figure 2: Schematic Representation of Two-Phase Flow Patterns In Adiabatic Vertical Upflow

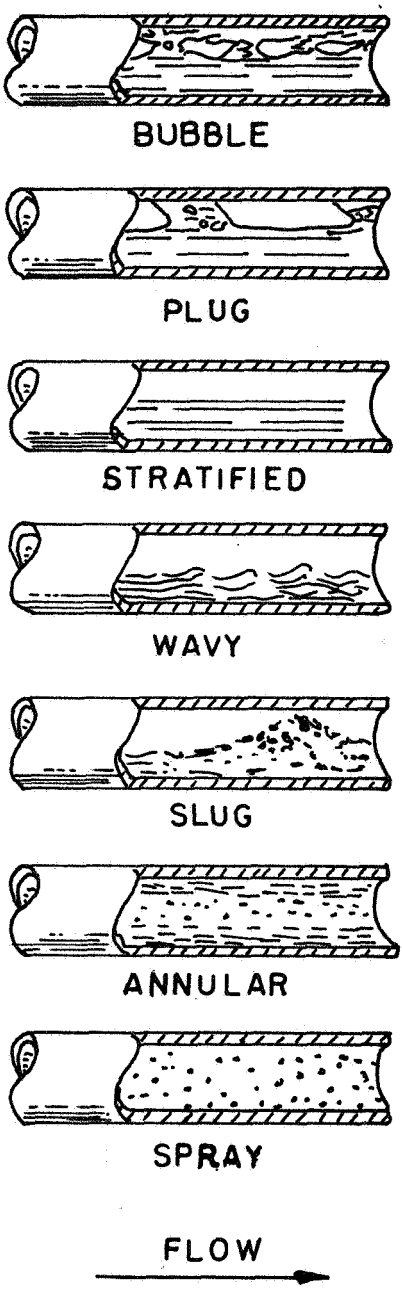


Figure 3: Schematic Representation of Two-Phase Flow Patterns in Adiabatic Horizontal Flow

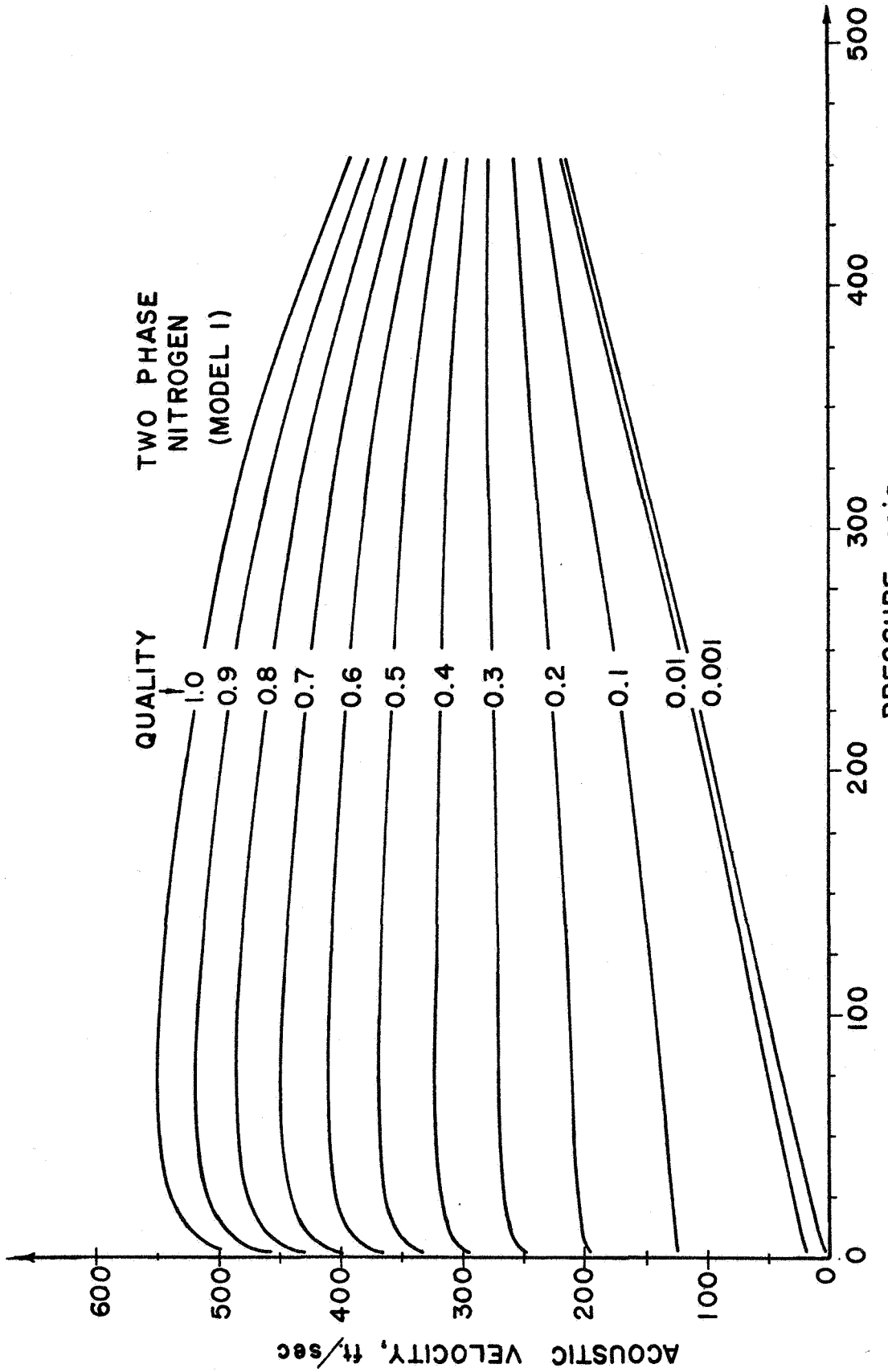


Figure 4: Acoustic Velocity as a Function of Saturation Pressure For Two-Phase Nitrogen.

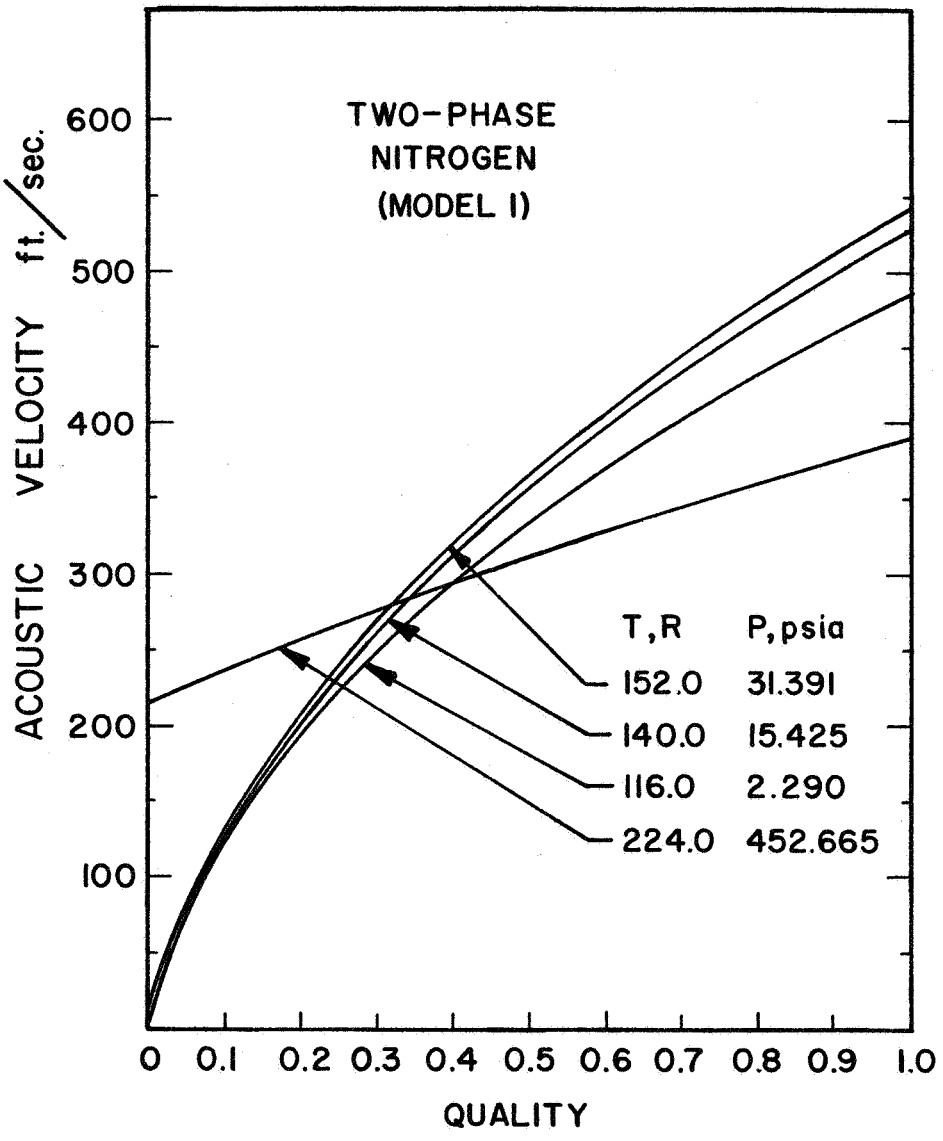


Figure 5: Acoustic Velocity as a Function of Quality for Two-Phase Nitrogen.

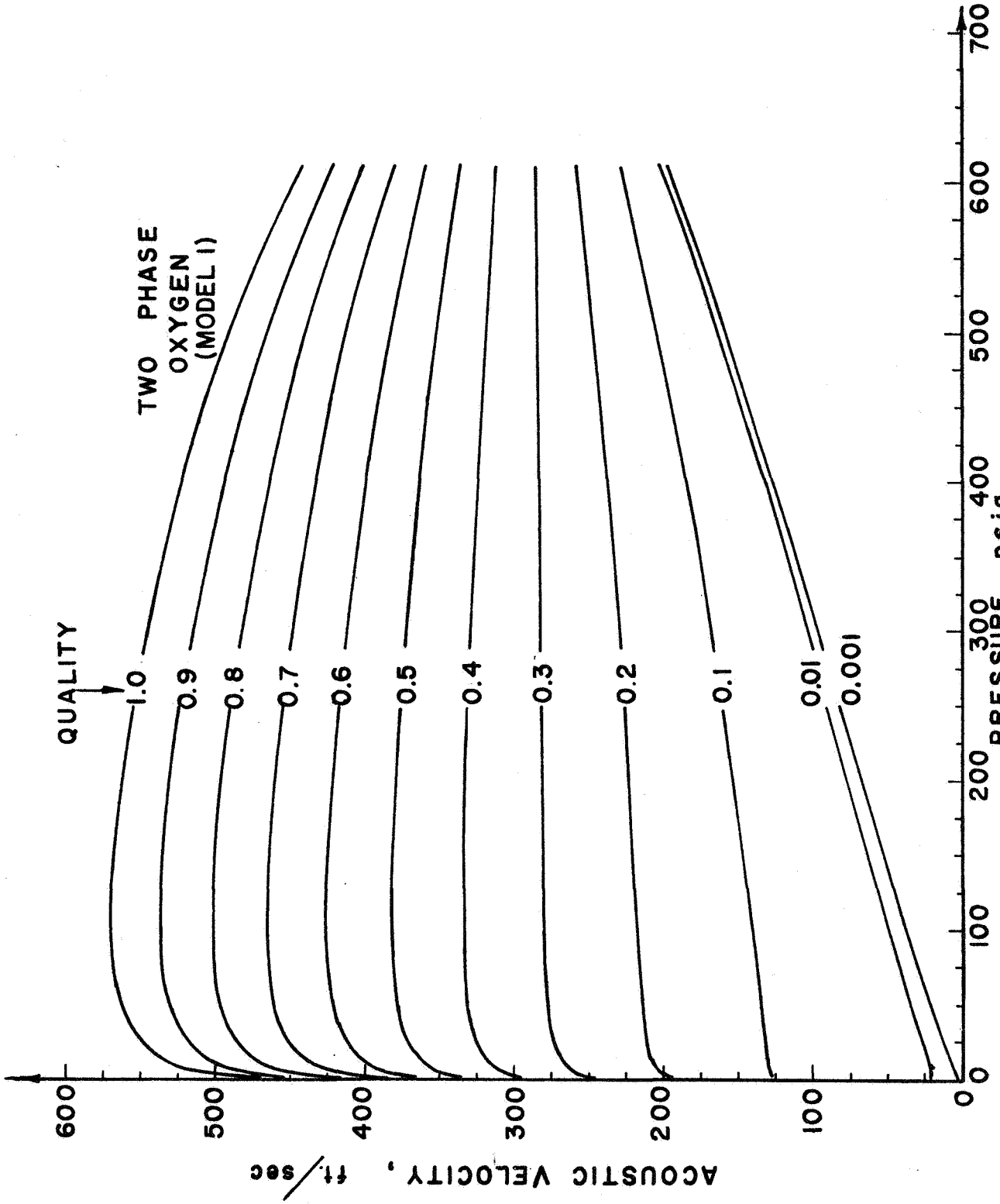


Figure 6: Acoustic Velocity as a Function of Saturation Pressure for Two-Phase Oxygen.

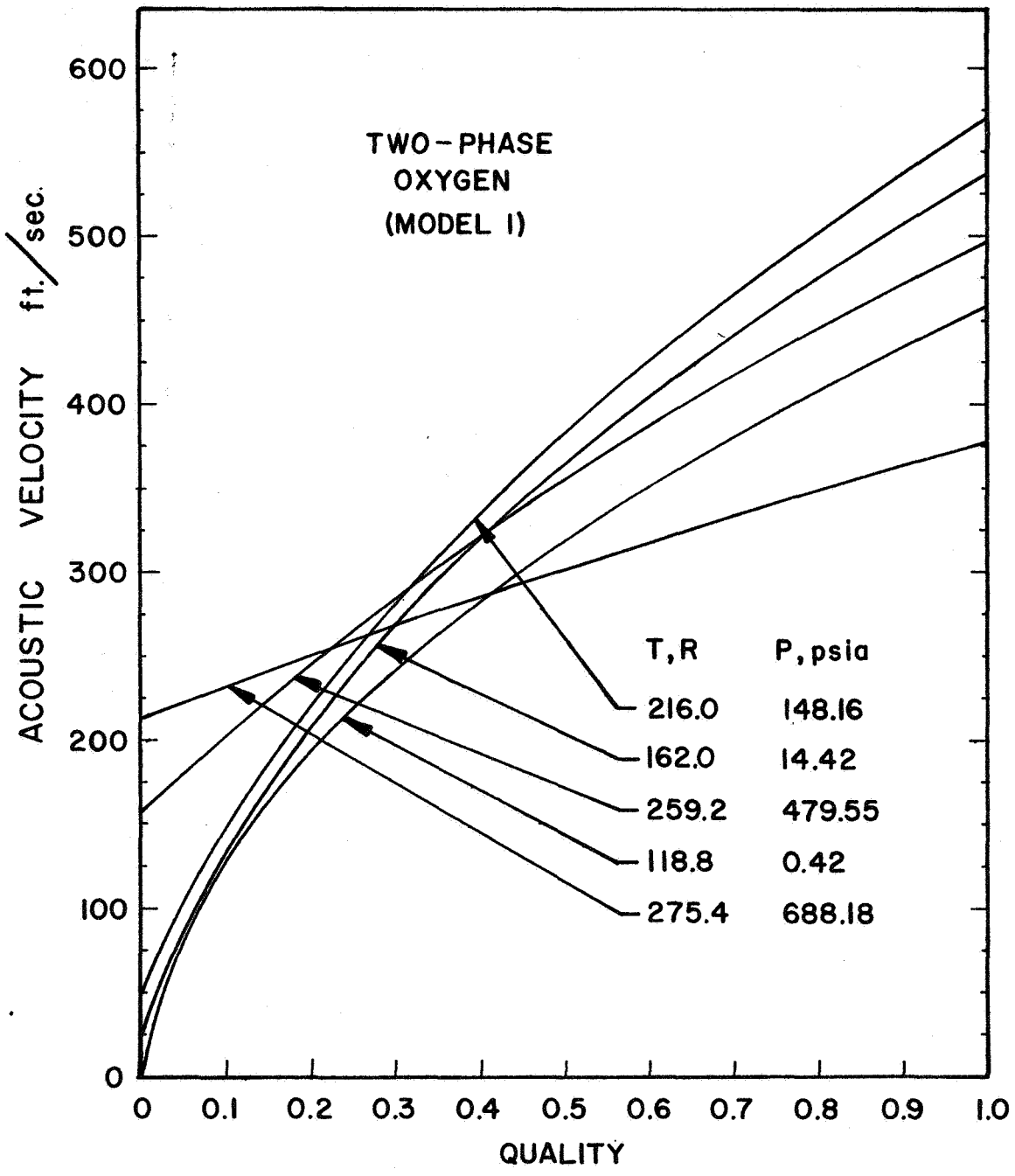


Figure 7: Acoustic Velocity as a Function of Quality for Two-Phase Oxygen.

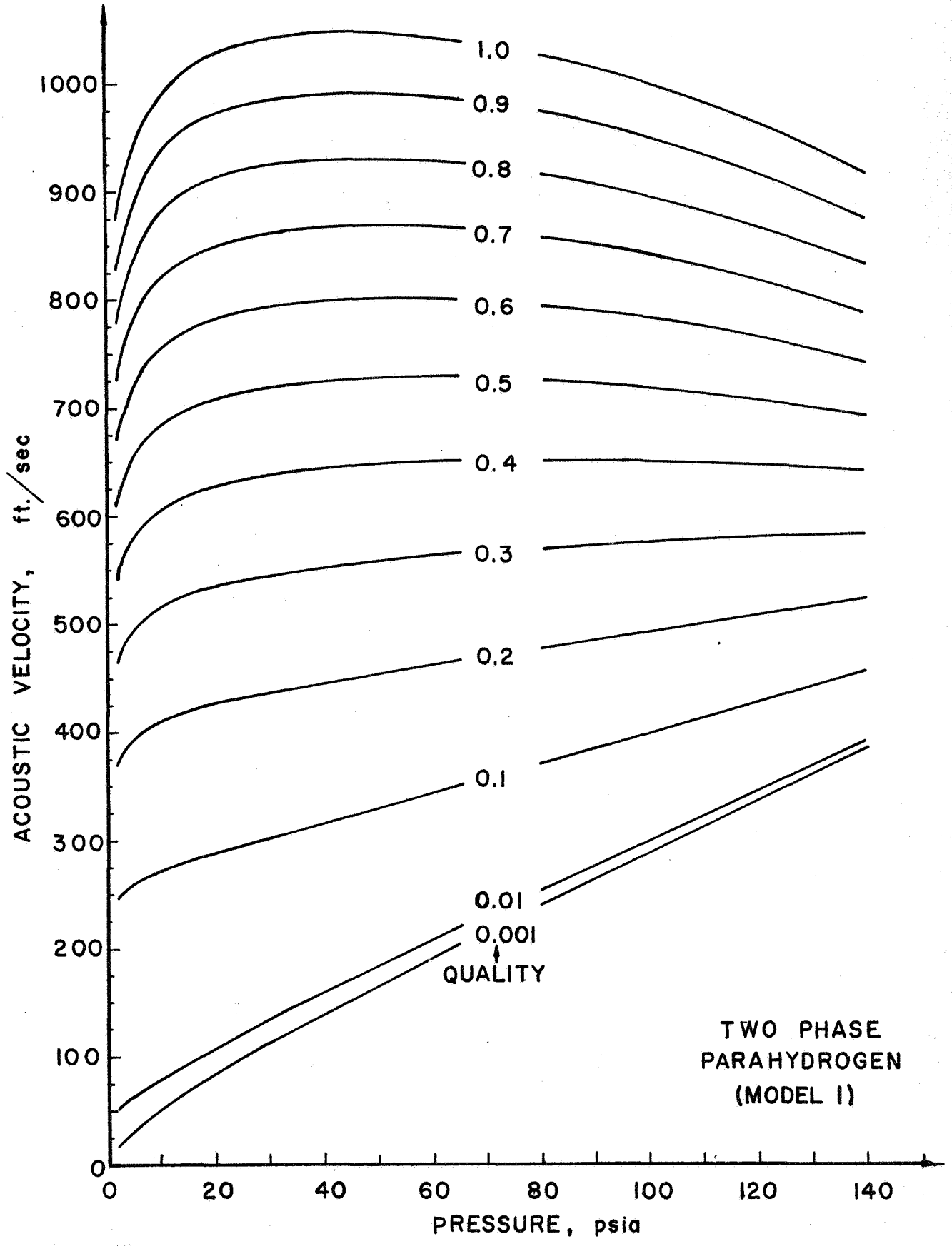


Figure 8: Acoustic Velocity as a Function of Saturation Pressure for Two-Phase Parahydrogen.

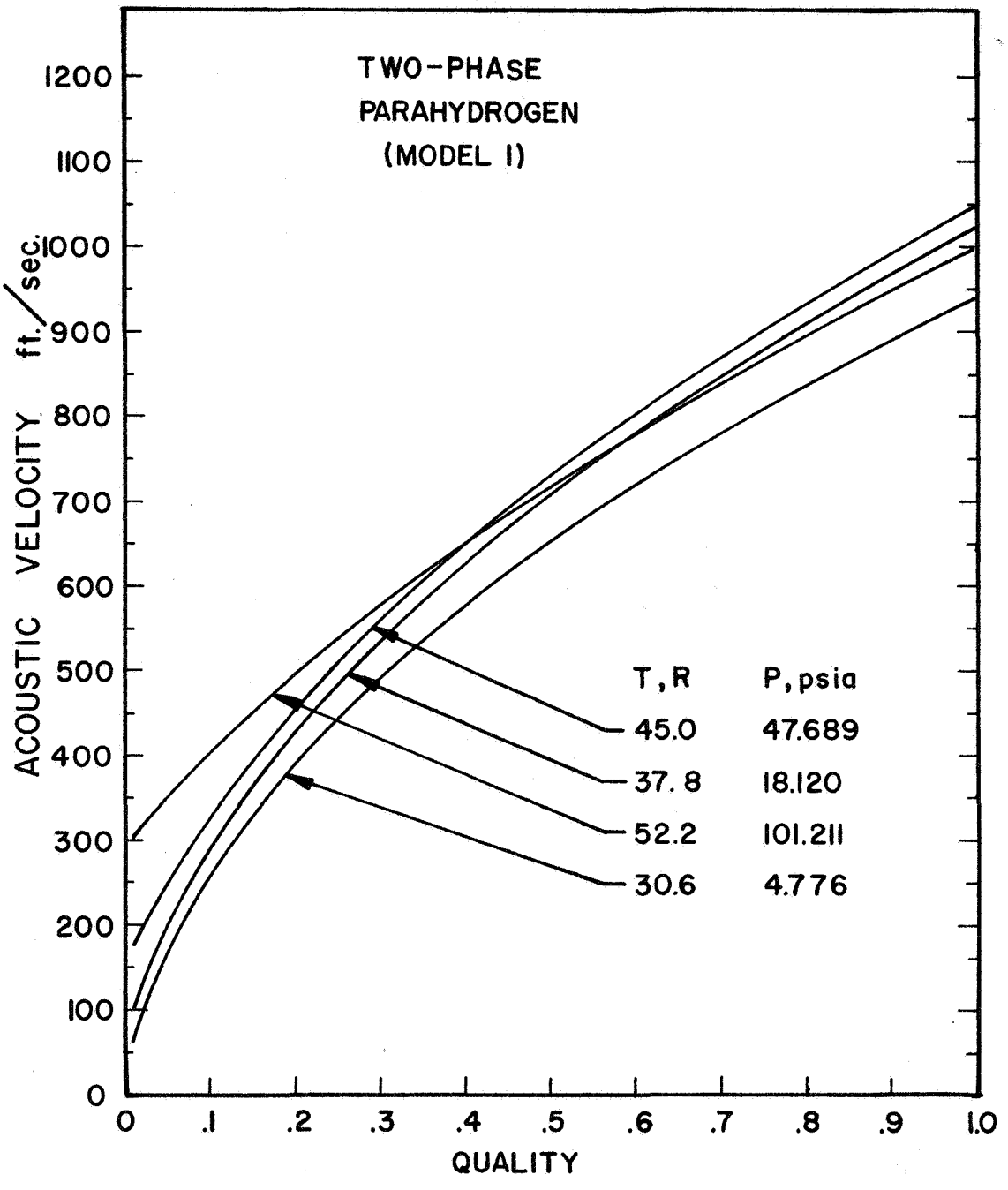


Figure 9: Acoustic Velocity as a Function of Quality for Two-Phase Parahydrogen.

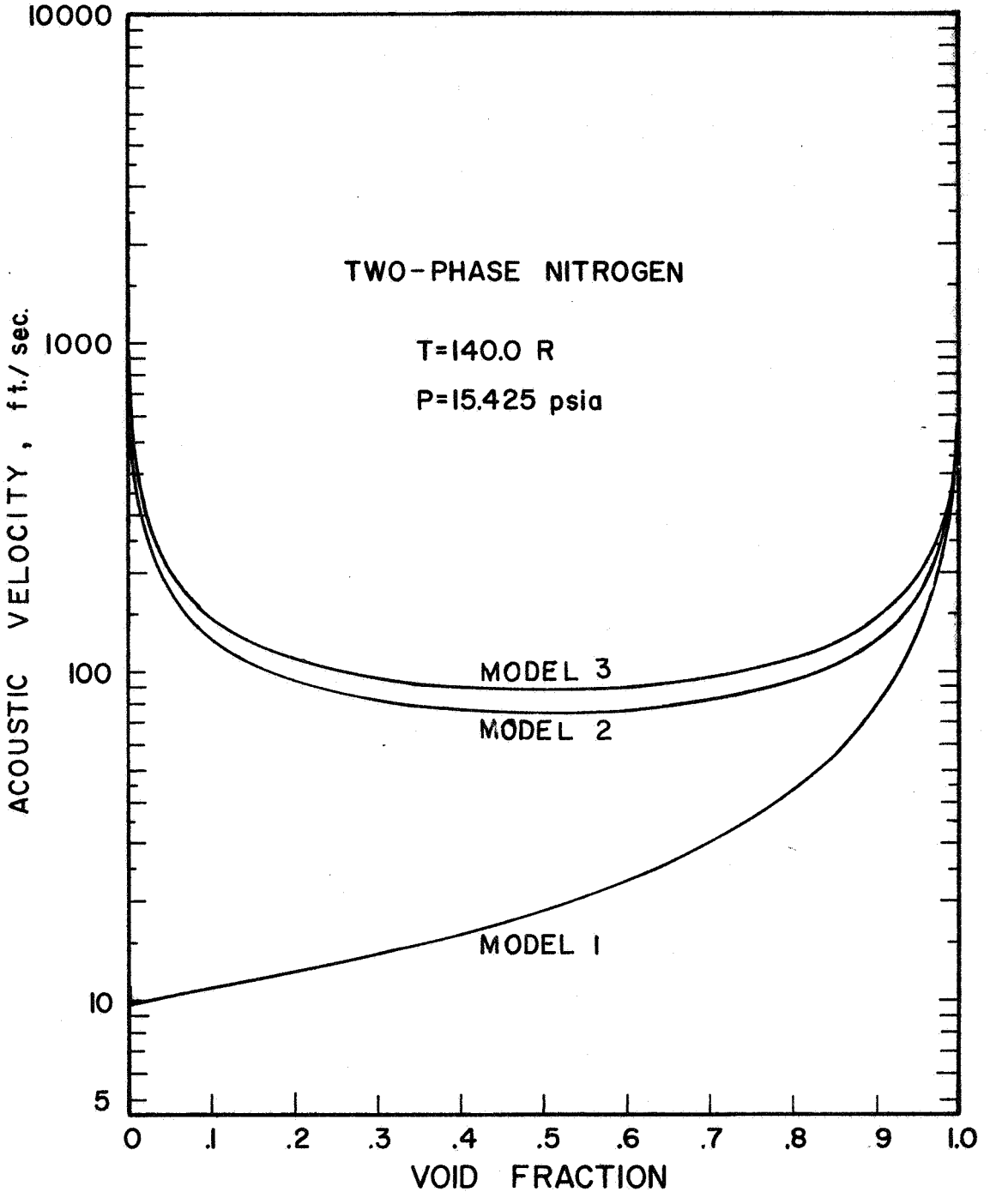


Figure 10: Comparison of Acoustic Velocity Models for Two-Phase Nitrogen

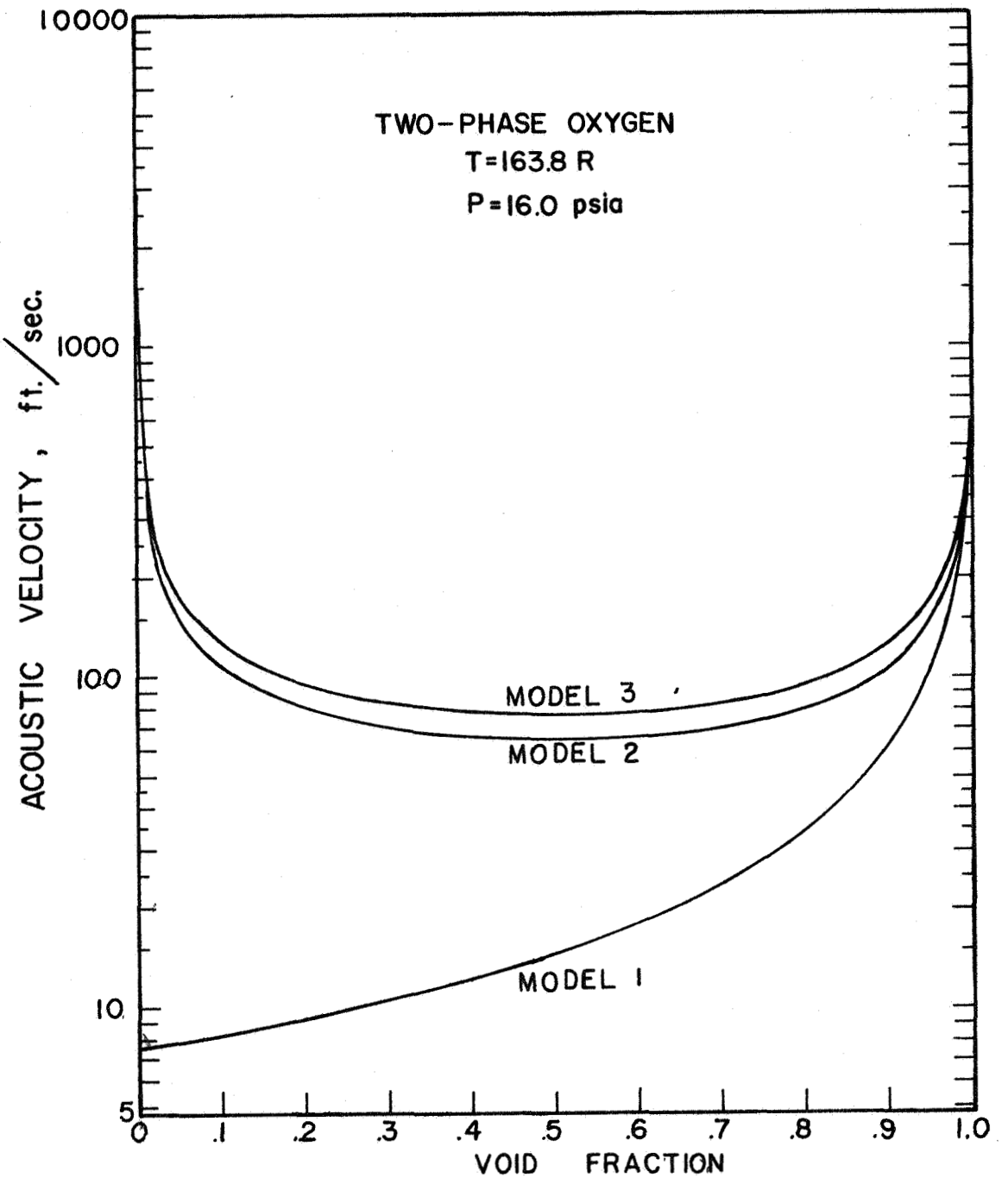


Figure 11: Comparison of Acoustic Velocity Models for Two-Phase Oxygen

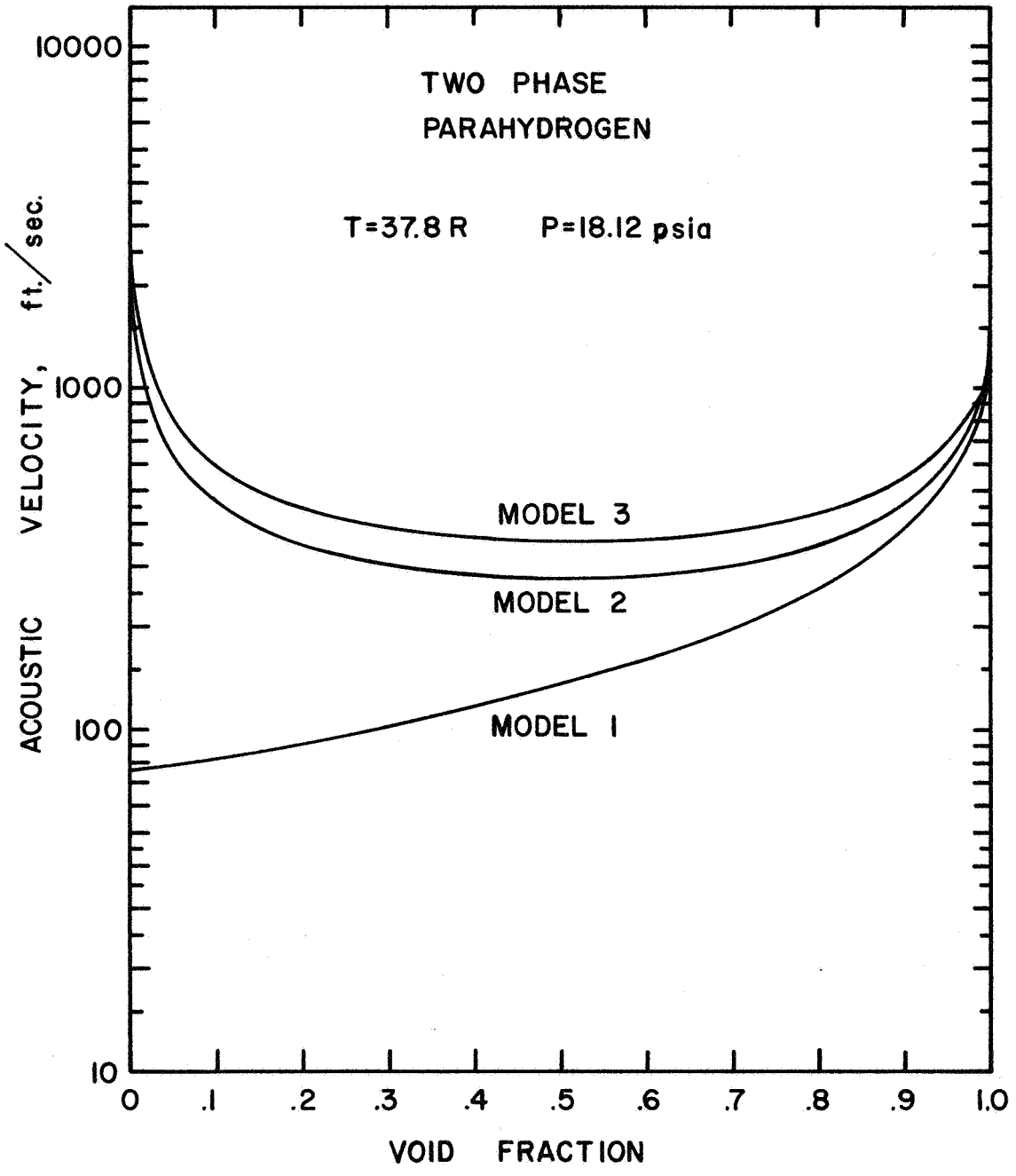


Figure 12: Comparison of Acoustic Velocity Models for Two-Phase Parahydrogen.

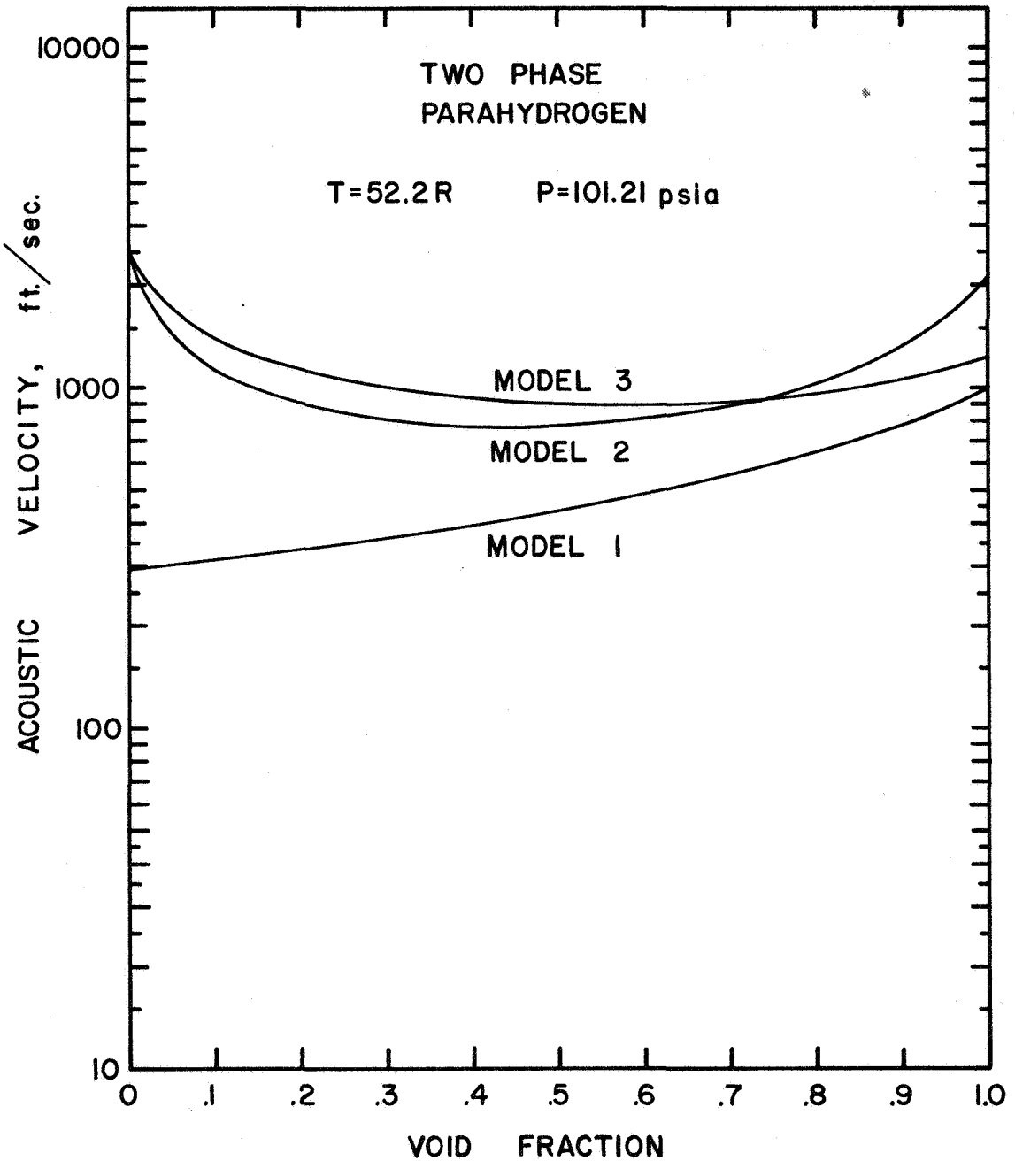


Figure 13: Comparison of Acoustic Velocity Models for Two-Phase Parahydrogen.

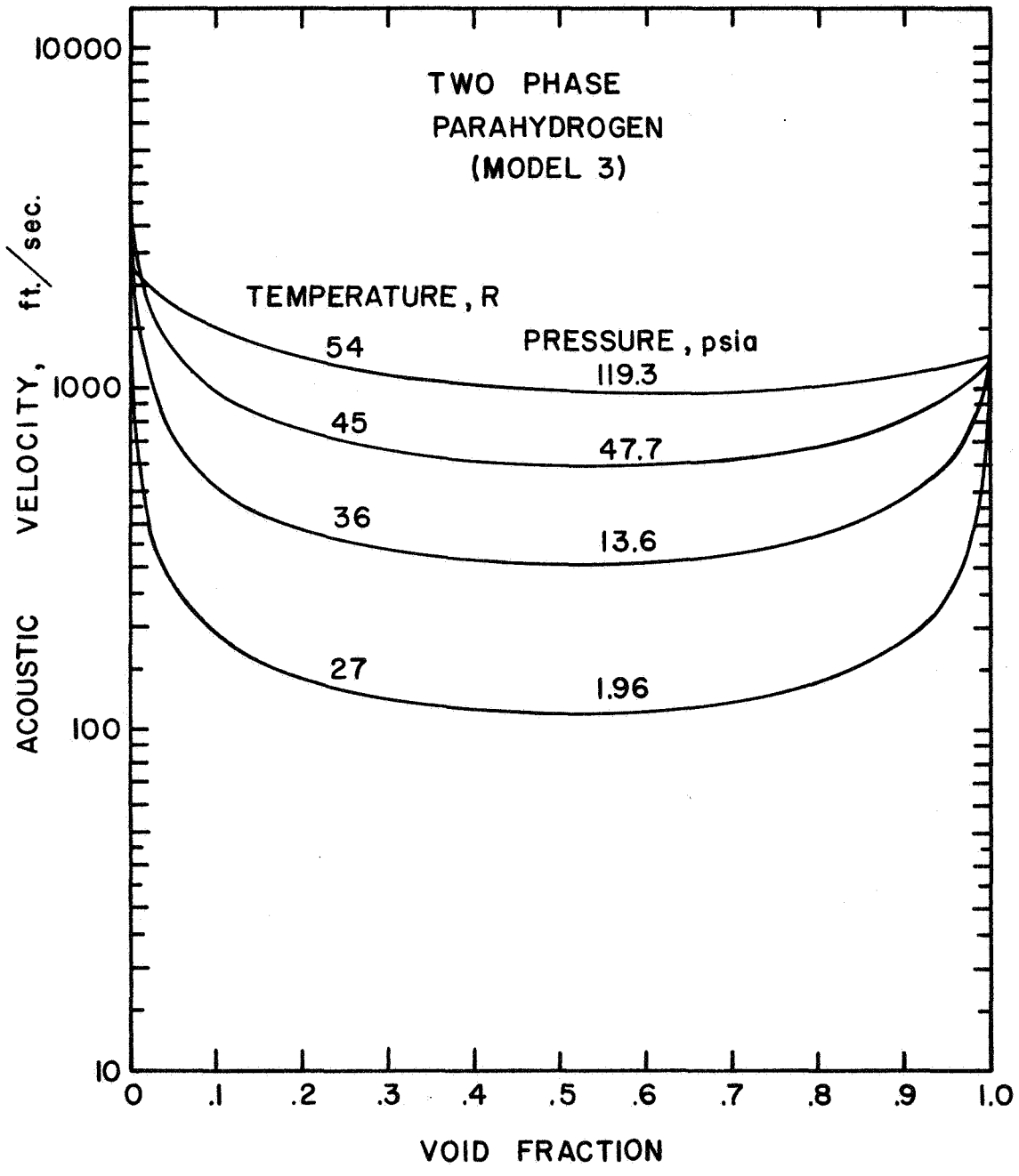


Figure 14: Effect of Saturation Pressure and Temperature on Acoustic Velocity Values by Model 3 for Parahydrogen.

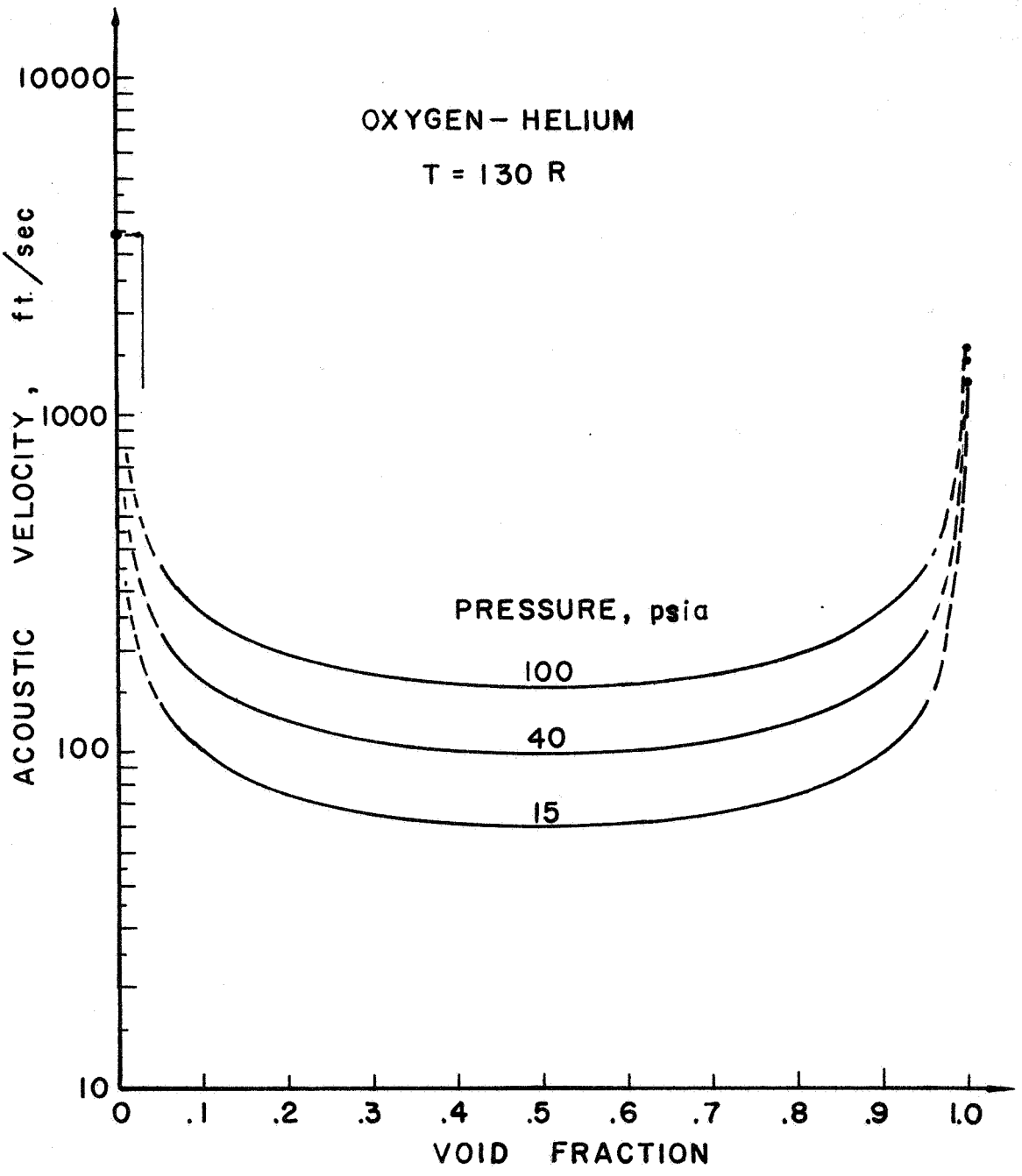


Figure 15: Acoustic Velocity as a Function of Void Fraction for Two-Phase Oxygen-Helium By Model 2.

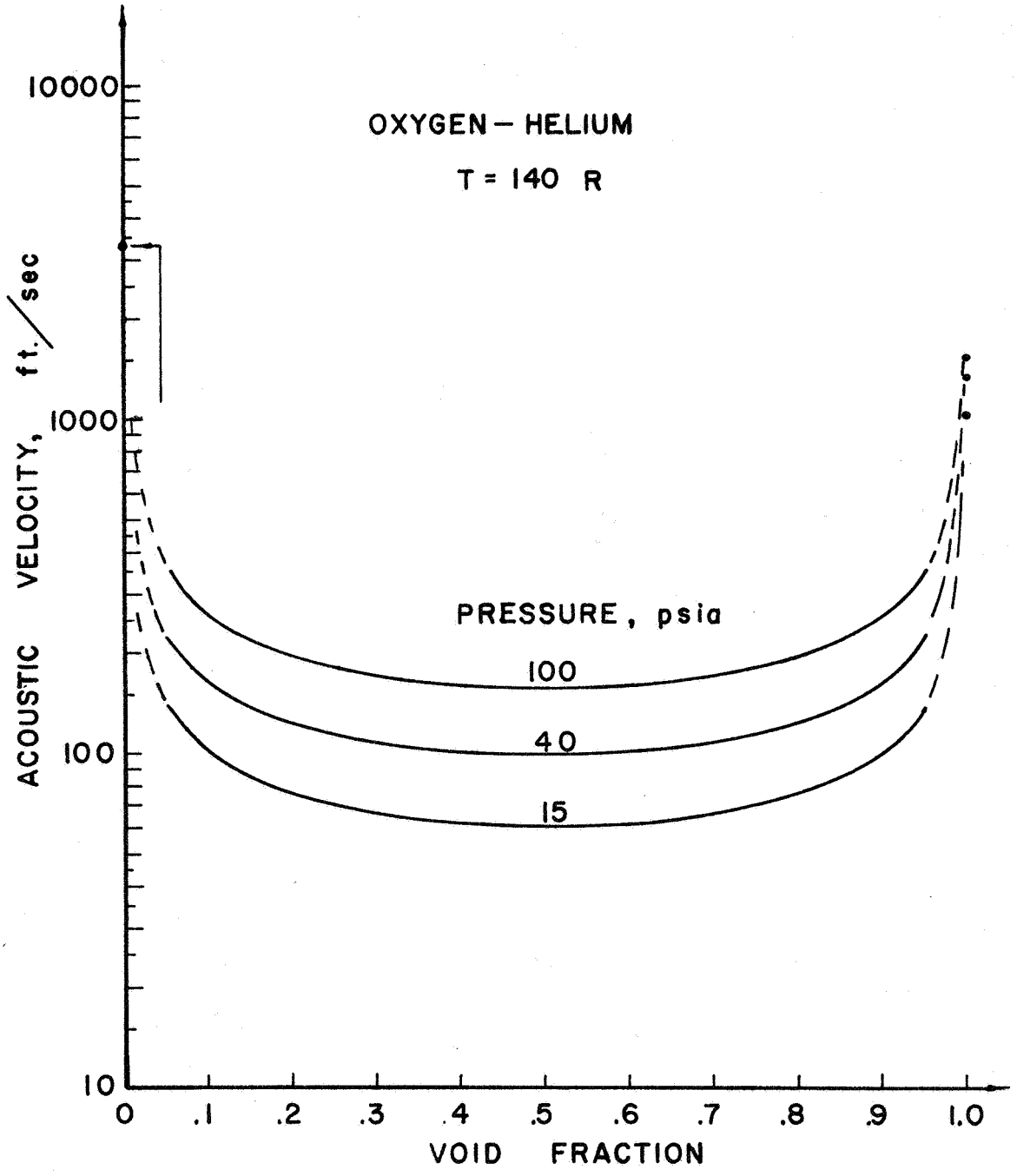


Figure 16: Acoustic Velocity as a Function of Void Fraction for Two-Phase Oxygen-Helium By Model 2.

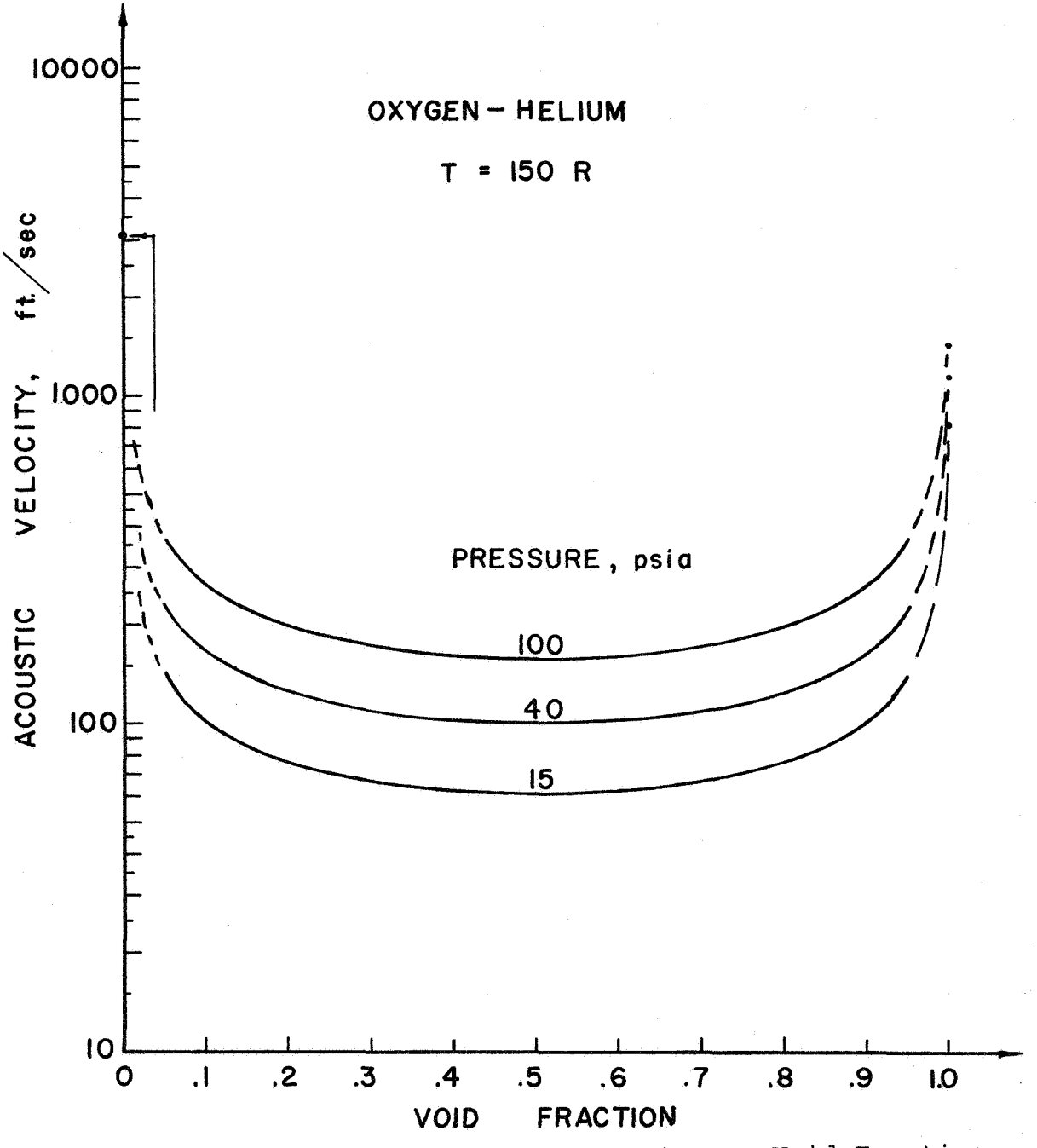


Figure 17: Acoustic Velocity as a Function of Void Fraction for Two-Phase Oxygen-Helium By Model 2.

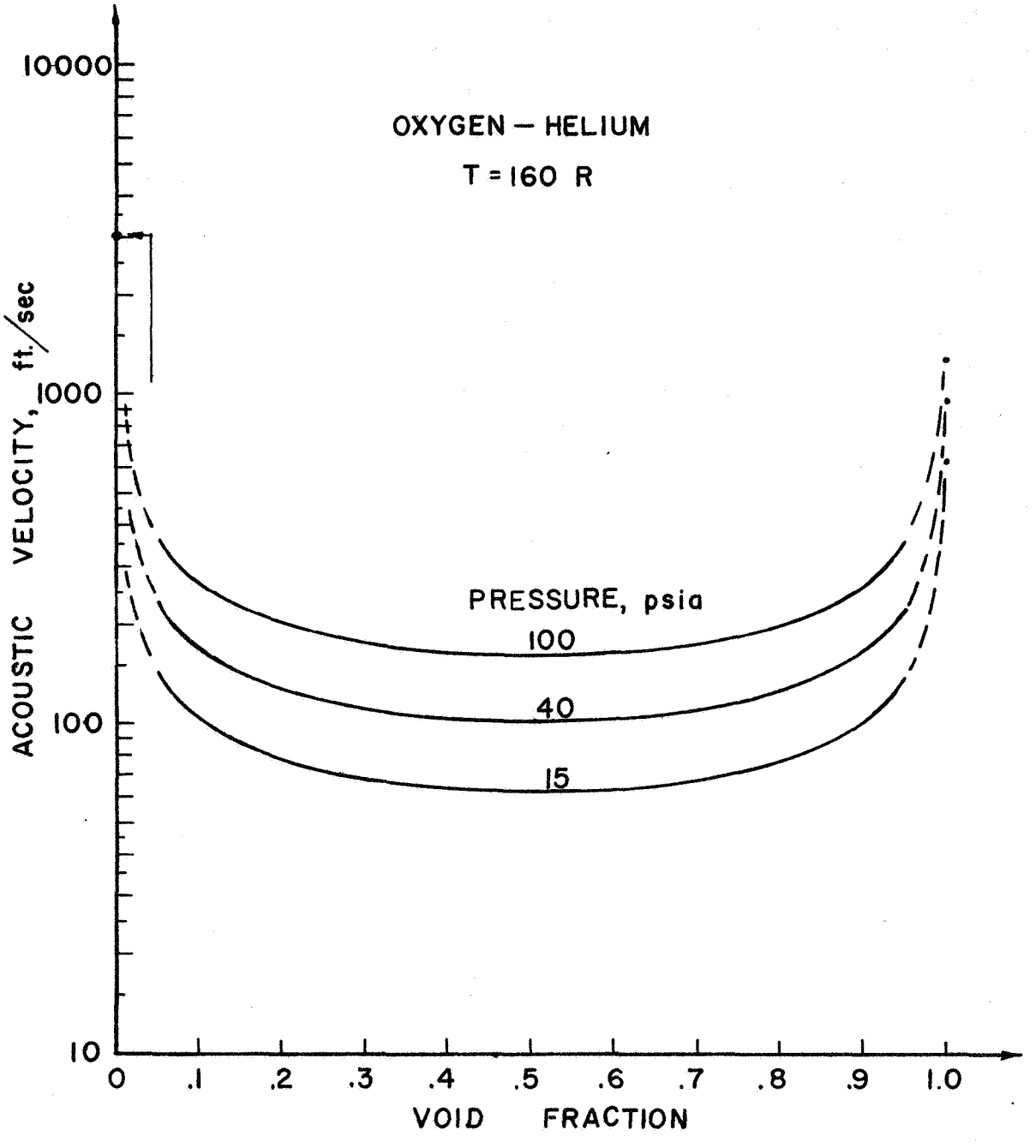


Figure 18: Acoustic Velocity as a Function of Void Fraction for Two-Phase Oxygen-Helium By Model 2.

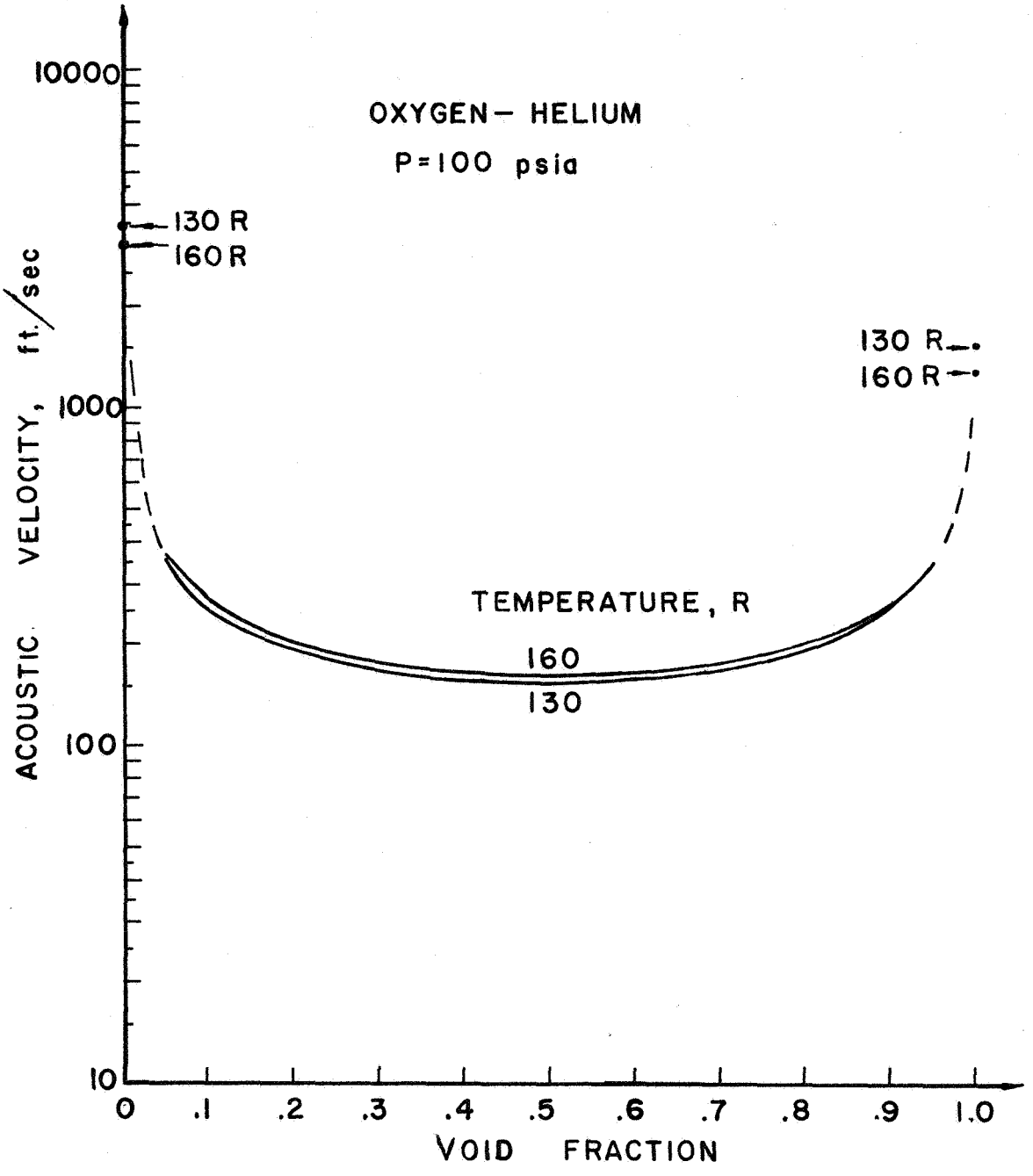


Figure 19: Effect of Temperature on Acoustic Velocities of Two-Phase Oxygen-Helium By Model 2.

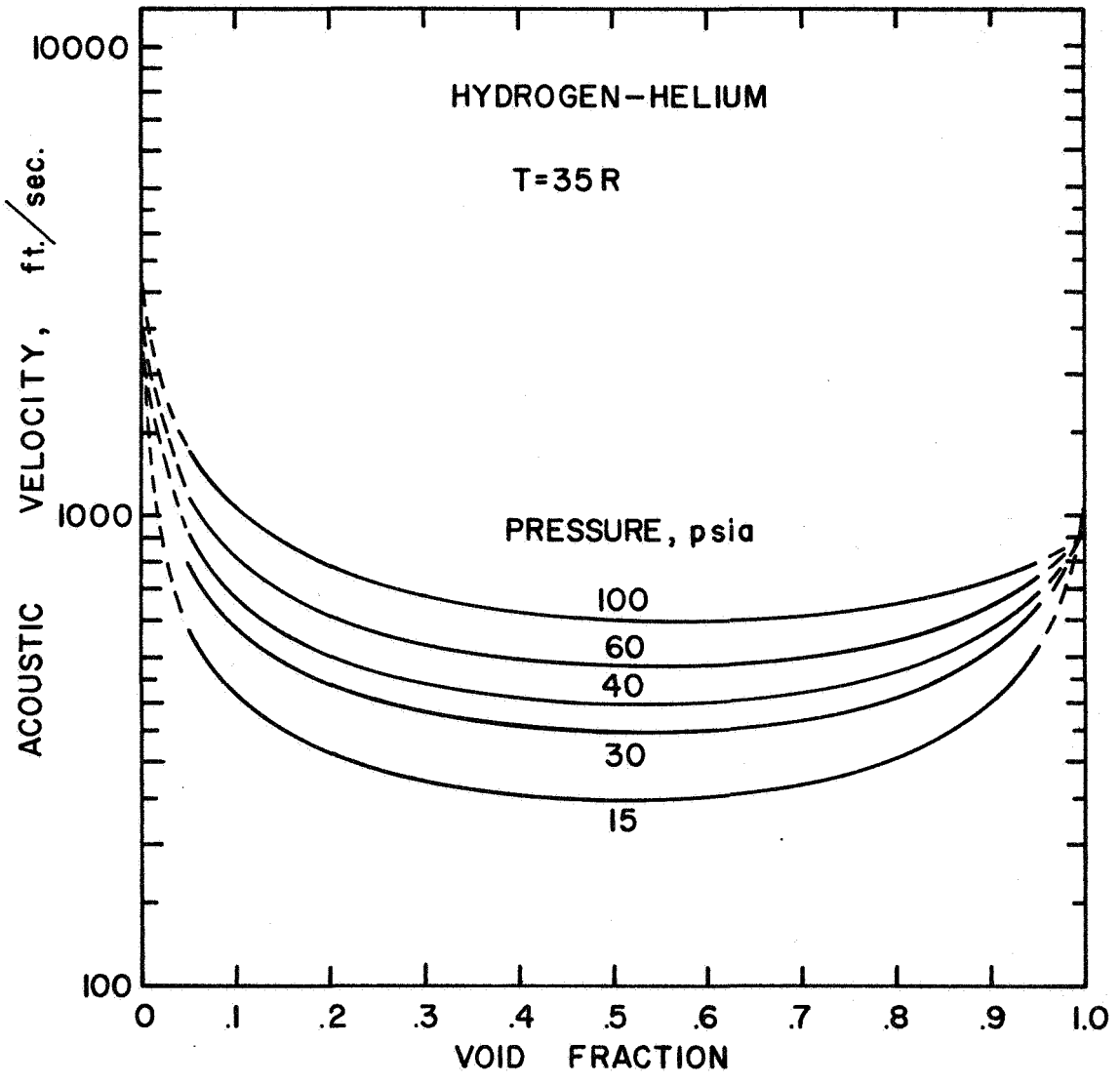


Figure 20: Acoustic Velocity as a Function of Void Fraction for Two-Phase Hydrogen-Helium By Model 2.

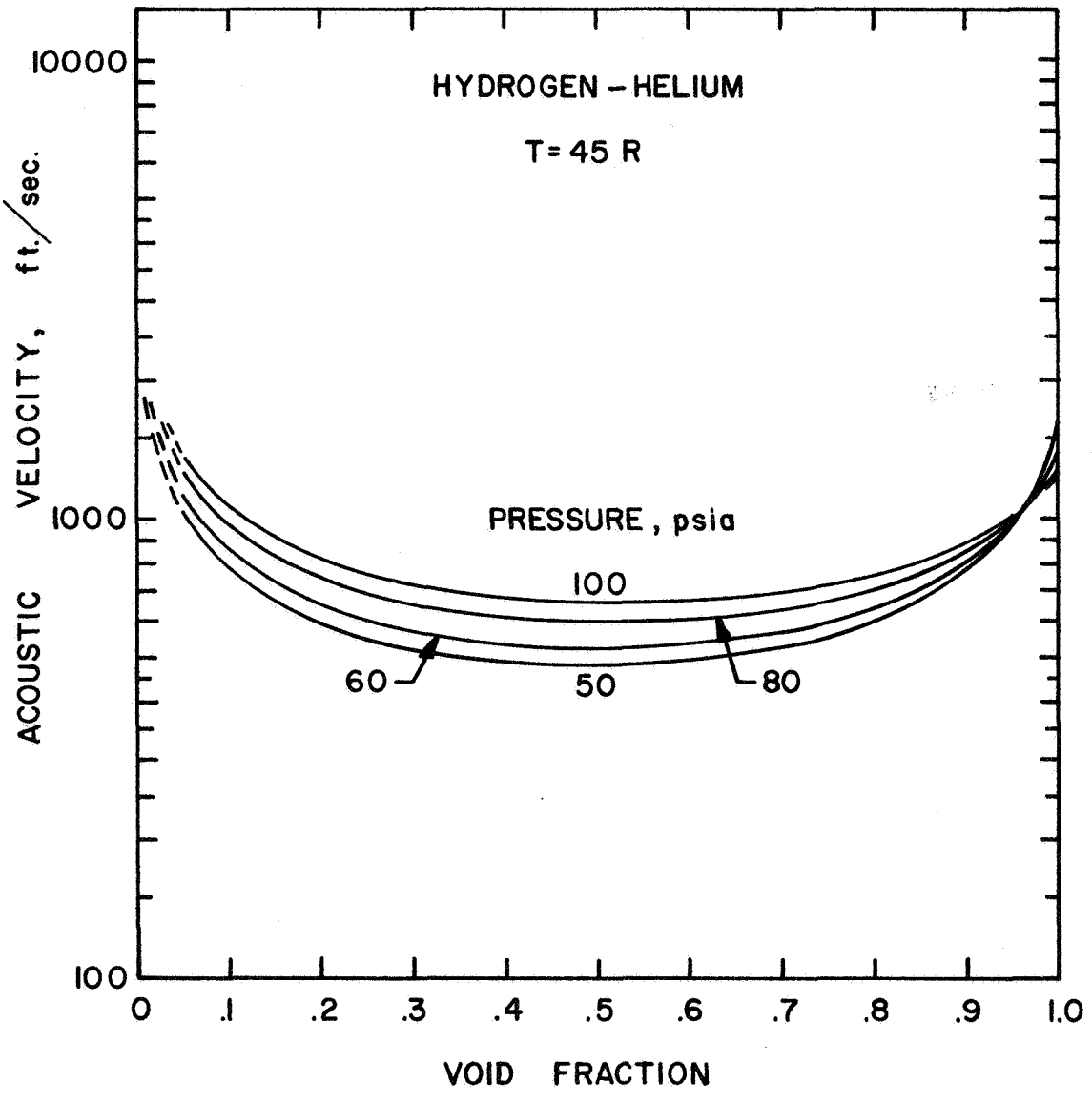


Figure 21: Acoustic Velocity as a Function of Void Fraction for Two-Phase Hydrogen-Helium By Model 2.

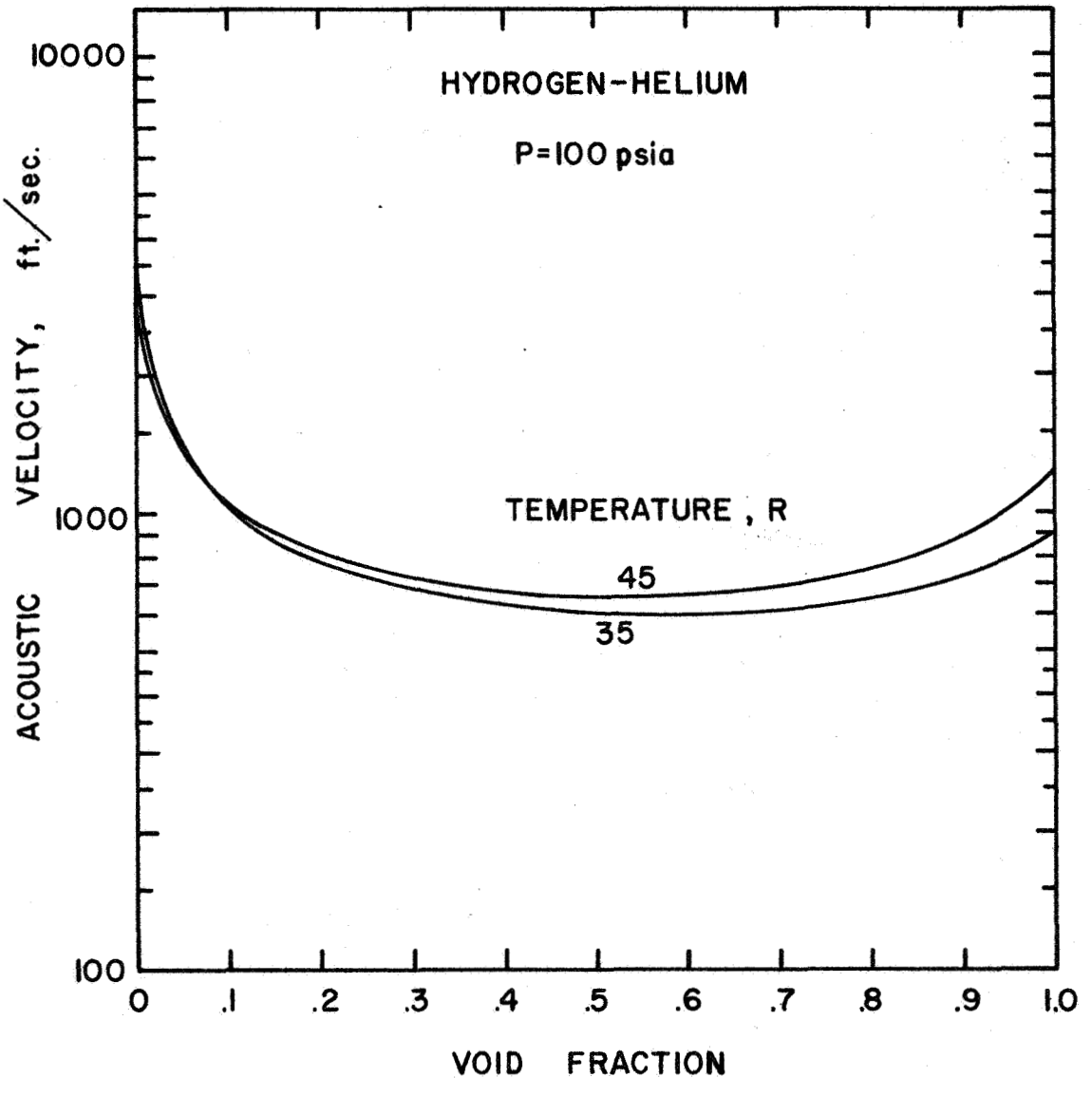


Figure 22: Effect of Temperature on Acoustic Velocities of Two-Phase Hydrogen-Helium By Model 2.

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12. Campbell, Hugh M., Jr., and Overcamp, Thomas J., Critical Flow Rate of Two-Phase Nitrogen, Proceedings of the Southeastern Symposium on Missiles and Aerospace Vehicles Sciences, American Astronautical Society, Vol. I, December 5, 6, 7, 1966, Huntsville, Alabama.

## APPENDIX

Derivations for the models used in the calculations of this report have been reported previously in the literature; however, they are reviewed here for internal consistency.

### Model 1

The derivation of equation (10) is reported in reference (6). Consider equation (6).

$$C^2 = -g_c v^2 \left(\frac{\partial P}{\partial v}\right)_s \tag{6}$$

For the pseudo fluid, v is replaced by the mean specific volume given by equation (4).

i.e.

$$C^2 = -g_c \bar{v}^2 \left(\frac{\partial P}{\partial \bar{v}}\right)_{\bar{s}} \tag{A-1}$$

where

$$\bar{v} = v_1 + x(v_2 - v_1) \tag{A-2}$$

Differentiation of equation (A-2) yields:

$$\left(\frac{\partial \bar{v}}{\partial P}\right)_{\bar{s}} = \frac{dv_1}{dP} + x\left(\frac{dv_2}{dP} - \frac{dv_1}{dP}\right) + (v_2 - v_1)\left(\frac{\partial x}{\partial P}\right)_{\bar{s}} \tag{A-3}$$

The derivatives of $v_1$ and $v_2$ are written as total derivatives since $v_1$ and $v_2$ are saturation values which can be expressed as functions only of the pressure. The entropy per lbm of the mixture is:

$$\bar{s} = s_1 + x(s_2 - s_1) \tag{A-4}$$

Differentiation of equation (A-4) with respect to pressure yields:

$$\frac{d\bar{s}}{dP} = \frac{ds_1}{dP} + x\left(\frac{ds_2}{dP} - \frac{ds_1}{dP}\right) + (s_2 - s_1)\frac{dx}{dP} \tag{A-5}$$

For an isentropic process $\bar{s}$ = constant. Then

$$\frac{d\bar{s}}{dP} = 0 \tag{A-6}$$

and equation (A-5) then gives:

$$\left(\frac{\partial x}{\partial P}\right)_{\bar{s}} = -\frac{1}{(s_2-s_1)}\frac{ds_1}{dP} - \frac{x}{(s_2-s_1)}\left(\frac{ds_2}{dP} - \frac{ds_1}{dP}\right) \tag{A-7}$$

Substitution of equation (A-7) into equation (A-3) gives:

$$\left(\frac{\partial \bar{v}}{\partial P}\right)_{\bar{s}} = \frac{dv_1}{dP} + x\left(\frac{dv_2}{dP} - \frac{dv_1}{dP}\right) - \frac{(v_2-v_1)}{(s_2-s_1)}\frac{ds_1}{dP} - x\frac{(v_2-v_1)}{(s_2-s_1)}\left(\frac{ds_2}{dP} - \frac{ds_1}{dP}\right) \tag{A-8}$$

Substitution of equation (A-8) into equation (A-1) gives:

$$C^2 = \frac{g_c\,[v_1 + x(v_2-v_1)]^2}{x\left[\frac{(v_2-v_1)}{(s_2-s_1)}\left(\frac{ds_2}{dP}-\frac{ds_1}{dP}\right)-\left(\frac{dv_2}{dP}-\frac{dv_1}{dP}\right)\right]+\frac{(v_2-v_1)}{(s_2-s_1)}\frac{ds_1}{dP}-\frac{dv_1}{dP}} \qquad (10)$$

Model 2

Equation (11) which was used for model 2 was derived in reference (2). some of the steps are included here for reference. Combination of equations (4) and (12) gives:

$$v_2 = \frac{1+\phi}{\phi}\,\bar{v} - \frac{1}{\phi}\,v_1 \qquad (A-9)$$

Thermodynamic property relations for phase 1 and phase 2 give:

$$ds_1 = C_{v1}dT + Pdv_1 \qquad (A-10)$$

$$ds_2 = C_{v_2}dT + Pdv_2 \qquad (A-11)$$

For an isentropic pressure disturbance,

$$d(ms) = m_1ds_1 + m_2ds_2 = 0 \qquad (A-12)$$

$$m_1C_{v_1}dT + m_1Pdv_1 + m_2Cv_2dT + m_2Pdv_2 = 0 \qquad (A-13)$$

or

$$(C_{v_1} + \phi C_{v_2})dT + Pdv_1 + \phi Pdv_2 = 0 \qquad \text{(A-14)}$$

since

$$\phi = \frac{m_2}{m_1} \qquad \text{(A-15)}$$

For convenience, an average specific heat is written as

$$\bar{C}_v = C_{v_1} + \phi C_{v_2} \qquad \text{(A-16)}$$

Therefore

$$\bar{C}_v dT + Pdv_1 + \phi Pdv_2 = 0 \qquad \text{(A-17)}$$

The ideal gas equation of state yields:

$$dT = T\left(\frac{dP}{P} + \frac{dv_2}{v_2}\right) \qquad \text{(A-18)}$$

Hence

$$\bar{C}_v \frac{T}{P} dP + Pdv_1 + \phi Pdv_2 + \bar{C}_v \frac{T}{v_2} dv_2 = 0 \qquad \text{(A-19)}$$

Differention of equation (A-9) assuming no phase change yields:

$$dv_2 = \frac{1+\phi}{\phi} d\bar{v} - \frac{1}{\phi} dv_1 \qquad \text{(A-20)}$$

Substitution of equation (A-20) into equation (A-19) gives:

$$\bar{C}_v \frac{dT}{P} dP + Pdv_1 + (\phi P + \frac{\bar{C}_v P}{R}) \left[\frac{1+\phi}{\phi} d\bar{v} - \frac{1}{\phi} dv_1\right] = 0 \qquad \text{(A-21)}$$

or

$$\bar{C}_v \frac{T}{P} dP - \frac{P}{R} \frac{\bar{C}_v}{\phi} dv_1 + P \left(\frac{\bar{C}_v}{R} + \phi\right)\left(\frac{1+\phi}{\phi}\right) d\bar{v} = 0 \qquad \text{(A-22)}$$

Equation (7) gives

$$dv_1 = -\beta_1 v_1 dP \qquad \text{(A-23)}$$

Therefore

$$\bar{C}_v \frac{T}{P} dP + \frac{P}{R} \frac{\bar{C}_v}{\phi} \beta_1 v_1 dP + P \left(\frac{\bar{C}_v}{R} + \phi\right)\left(\frac{1+\phi}{\phi}\right) d\bar{v} = 0 \qquad \text{(A-24)}$$

Then

$$\left(\frac{\partial P}{\partial \bar{v}}\right)_s = \frac{-\frac{P}{R}(\bar{C}_v + R\phi)\left(\frac{1+\phi}{\phi}\right)}{\bar{C}_v \frac{T}{P} + \frac{P}{R}\frac{\bar{C}_v}{\phi}\beta_1 v_1} \qquad \text{(A-25)}$$

or

$$\left(\frac{\partial P}{\partial \bar{v}}\right)_s = -\frac{(\rho_2^2 RT)(\bar{C}_v + R\phi)(1+\phi)}{(\bar{C}_v \phi)\left(1 + \frac{P\beta_1\rho_2}{\phi\rho_1}\right)} \qquad \text{(A-26)}$$

Substitution of equation (A-26) into equation (6) yields:

$$c^2 = \frac{g_c \bar{v}^2 \rho_2^2 RT (\bar{C}_v + R\phi)(1+\phi)}{(\bar{C}_v \phi)\left(1 + \frac{P\beta_1\rho_2}{\phi\rho_1}\right)} \qquad \text{(A-27)}$$

Equation (9) is used to form a ratio:

$$\frac{c^2}{c_2^2} = \frac{\bar{v}^2 \rho_2^2 (\bar{C}_v + R\phi)(1+\phi)}{(\bar{C}_v \phi K_2)\left(1 + \frac{P\beta_1\rho_2}{\phi\rho_1}\right)} \qquad \text{(A-28)}$$

Equation (A-9) yields:

$$\bar{v} = \frac{1 + \phi \frac{\rho_1}{\rho_2}}{\rho_1(1 + \phi)} \tag{A-29}$$

Then

$$\left(\frac{C}{C_2}\right)^2 = \frac{(\rho_2/\rho_1 + \phi)^2 \left(\frac{C_{v_1}}{K_2 C_{v_2}} + \phi\right)}{\phi(1+\phi)\left(\frac{C_{v_1}}{C_{v_2}} + \phi\right)\left(1 + \frac{P\beta_1\rho_2}{\phi\rho_1}\right)} \tag{11}$$

Model 3

Equations (4), (5), and (6) are:

$$\bar{v} = xv_2 + (1-x)v_1 \tag{4}$$

$$\bar{\rho} = \alpha\rho_2 + (1-\alpha)\rho_1 \tag{5}$$

$$C^2 = -g_c v^2 \left(\frac{\partial P}{\partial v}\right)_s \tag{6}$$

Differentiation of equation (4) assuming no phase change gives:

$$\left(\frac{\partial \bar{v}}{\partial P}\right)_{\bar{s}} = x\left(\frac{\partial v_2}{\partial P}\right)_{\bar{s}} + (1-x)\left(\frac{\partial v_1}{\partial P}\right)_{\bar{s}} \tag{A-30}$$

One plausible interpretation of the derivatives on the right-hand side of equation (A-30) is:

$$\left(\frac{\partial v_2}{\partial P}\right)_{\bar{s}} \simeq \frac{-g_c v_2^2}{C_2^2} \tag{A-31}$$

and

$$\left(\frac{\partial v_1}{\partial P}\right)_{\bar{s}} \simeq \frac{-g_c v_1^2}{C_1^2} \tag{A-32}$$

Then

$$C^2 = \frac{g_c \bar{v}^2}{g_c[x\, v_2^2/C_2^2 + (1-x) v_1^2/C_1^2]} \tag{A-33}$$

or

$$\frac{1}{C^2} = \bar{\rho}^2 \left[\frac{x v_2}{C_2^2 \rho_2} + \frac{(1-x) v_1}{C_1^2 \rho_1}\right] \tag{A-34}$$

Void fraction and quality are related by:

$$x\bar{\rho} = \alpha \rho_2 \tag{A-35}$$

$$(1-x)\bar{\rho} = (1-\alpha)\rho_1 \tag{A-36}$$

Then combination of equations (A-34), (A-35), and (A-36) yields:

$$\frac{1}{C^2} = [\alpha \rho_2 + (1-\alpha)\rho_1]\left[\frac{\alpha}{C_2^2 \rho_2} + \frac{1-\alpha}{C_1^2 \rho_1^2}\right] \tag{13}$$